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Report on the Habilitation Thesis of Jan Konecny entitled

Formal concept analysis with graded affirmations and denials

The general topic of the thesis concerns the extension of formal concept analysis to object-attribute data tables (formal contexts) with non-Boolean entries, a topic investigated by Radim Belohlavek, among others, since the mid-1990's. Mathematically, numerical data tables are supposedly encoded as fuzzy relations between objects and attributes, with values in a residuated lattice. A value in a formal context is either understood as a lower bound on some attribute value of an object (what is interpreted as a graded affirmation) or like an upper bound on this value (what is interpreted as a graded denial).

The key-notion in this investigation is that of a concept to be extracted from the data table. If the fuzzy relation is interpreted positively as containing affirmations, the mathematical tool for extracting concept is that of an antitone Galois connection. If it is interpreted negatively the mathematical tool is an isotone Galois connection. The originality of Jan Konecny research is an extensive investigation of graded isotone Galois connections and the extraction of concepts based on denials.

The thesis manuscript contains one introductory section giving the state of the art on the topic of formal concept analysis with graded data, and a second section containing a copy of 8 journal papers co-authored by the candidate, each preceded by a one-page presentation.

The introductory part contains material on complete residuated lattice theory, Zadeh's representation of hedges cast in the latter setting, fuzzy sets and relations with membership grades in a residuated lattice. Then the author reviews material on the extension of formal concept analysis with contexts representing graded data valued on complete residuated lattices: Galois connections, closures and interiors, ordered sets, concept lattices including hedges, and attribute implications extracted from the data and relating groups of attributes.

The topic is clearly appropriate for a habilitation thesis in computer science, and the above introductory part clearly accounts for the current state of knowledge.

The eight articles (out of 19 journal papers) published jointly with co-authors, between 2011 and 2017 and selected for the thesis contain the major contribution of Jan Koniecny. They correspond for the most part to research done after the author Ph.D. thesis. They can be described as follows

1. The development and study of antitone and isotone Galois connections with truth-stressing or truth-depressing hedges. As attributes of objects are valued on a lattice, they can be possessed by objects to a degree. Hedges can control the extent to which objects must satisfy attributes in order to appear in the extension of a concept. Antitone connections are instrumental to find sets of objects possessing attributes at least to some extent and isotone connections to find sets of objects possessing attributes

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at most to some extent. They are affirmations and denials that appear in the title of the thesis. The use of hedges enables the size of the concept lattices induced by the fuzzy context to be controlled.

2. The second paper focuses on the extraction of attribute implications from graded data. Two kinds of rules are extracted: some of the form " if some attributes have greater values than some thresholds then other attributes have greater values than other thresholds" and the other ones of the form " if some attributes have values less than some thresholds then other attributes have values less than other thresholds". A theoretical study of these rules and their connections with interior and closures, as well as a definition of semantic entailment of rules. It leads to a logic of dependencies in the style of Armstrong-like axioms.

3. The third paper starts from the remark that in the Boolean case antitone and isotone connections are closely related to each other via a simple transformation (exchanging zeros and ones in the context matrix). However, there is no longer this simple transformation in the graded case of residuated lattices because the negation may fail to be involutive. So in the graded case, the two kinds of Galois connections must be studied separately. In the paper the authors show how to recover part of the transformation by replacing the binary fuzzy relation by a ternary one involving membership grades as a third dimension, and by generalizing the usual residuated negation.

4. The fourth paper articulates affirmations and denials in the same formal context, represented by a bipolar fuzzy relation, whereby objects are valued by intervals in the residuated lattice. The intuition captured is that the extent to which each object possesses an attribute lies in an interval. The bottom of the interval represents an affirmation and the top a denial. The authors develop the apparatus of Galois connections and concept formation in this setting, using isotone and antitone relations conjointly. Likewise they develop the theory of attribute implications of the form « if some attributes have values in some intervals then other attributes have values in some other intervals ».

5. The fifth paper reconsiders formal contexts with affirmations and denials in terms of lower and upper approximations in the setting of fuzzy rough sets. The lower (resp. upper) membership grade of an object to an attribute then evaluates the extent to which it is certain (resp. possible) that the objects satisfy the properties expressed by the attributes. Concepts extracted from such rough contexts are themselves described by upper and lower approximations. Moreover they managed to use graded equivalence relations among attributes induced by the data-table to reduce the size of the rough concept lattice.

6. The sixth paper exploits analogies between fuzzy tolerance relations and fuzzy formal contexts, showing that the similarity classes of a fuzzy tolerance relation (reflexive and symmetric) correspond to fuzzy formal concepts in the sense of an extensive isotone fuzzy Galois connection, and can be partially ordered accordingly.

7. The seventh article is devoted to the generalization of the concept of block relation to the graded setting. The block relation notion is aimed at

clustering concepts in the concept lattice so as to reduce its size. It defines a symmetric and reflexive relation over concepts. The authors extend this notion to graded formal contexts using one antitone Galois connection and two isotone connections.

8. The last paper deals with the notion of bond between formal contexts. It enables to relate two distinct formal concepts showing dependencies between objects and attributes of the two formal contexts. The theory of bonds initiated by Wille is generalized to the graded context for three types of Galois connections.

Overall the set of papers presented (which represent only a part of published papers by the candidate in the same period) constitutes a clear break-through in the theory of formal context analysis, and can be useful for extracting knowledge from data-tables that are more general than the usual Boolean ones. From the mathematical point of view the paper relies on a strong algebraic framework and the obtained results are clearly original and non-trivial. These papers are published in well-known journals such as Information Sciences, Int. Approximate Reasoning, Fuzzy Sets and Systems, etc. The contents of the papers show that the candidate is a fine expert of data analysis using formal concept analysis and he masters the corresponding algebraic background. The content of the dissertation is certainly a major contribution to the foundations of the area of data science.

One would have liked to have more comments on the potential applicability of the approach (even if some of the papers contain practical examples, like in the topic of recommender systems for the choice of movies). Especially, do the assumption of a single value scale for all attributes and the choice of the residuated lattice structure restrict the applicability of the approach? Moreover one may have expected some pages describing the perspectives opened by these deep results and an outline of the future research plans by the candidate. These items appear to some extent in the conclusions of the above papers, but it would have been better to summarize them as a special short section of the document. These points can be topics for a discussion during the defense, and do not question the quantity of the work done by the candidate nor its scientific value.

All in all, I consider the habilitation work and the scientific achievements of Jan Koniecny until now fully meet the conditions requested, leading to the associate professor appointment.

January 24, 2019

Didier Dubois

Directeur de Recherche au CNRS