



Center for Machine Perception
Czech Technical University

Mirko Navara
Center for Machine Perception
Department of Cybernetics
Faculty of Electrical Engineering
Czech Technical University
Technická 2, 16627 Prague 6
Czech Republic

Phone: +420 2 2435 7388
Fax: +420 2 2435 7385
E-mail: navara@cmp.felk.cvut.cz

Referee's report on the habilitation thesis

Jan Konečný: Formal Concept Analysis with Graded Affirmations and Denials

Jan Konečný, Ph.D., is a well-known expert in formal concept analysis. Therefore I am not surprised that – instead of writing a separate habilitation thesis – he submitted a commented collection of his papers. Many of them were published in the leading journals in the field.

The high scientific level is already documented by the acceptance of the papers in high-quality journals; I can only acknowledge it. The introductory part gives a concise description of the state-of-the-art and the candidate's work in the field. I have read it with interest. I appreciated that also the contribution of each included paper is briefly and clearly explained.

The collection of papers documents the author's work in methods of formal concept analysis. Except for its classical (two-valued) version, its fuzzification is studied. This is quite natural because the relations of objects and attributes are often imprecise and can be better described by tools of multi-valued (fuzzy) logic. This theme is topical, as documented by numerous papers and conferences devoted to it. A problem arises with *negative information* (if we get evidence that an object *does not have* the attribute). Expressing it by another, negative attribute seems not a good choice (see my question 1 below). Thus a natural requirement is to extend the study by expressing *denials* and incorporating this to fuzzy concept analysis. This is the main topic of the thesis. The author brings a significant and interesting contribution, based on a generalization of fuzzy concept analysis which admits expressing *graded affirmations and graded denials*. Moreover, the truth values are not limited to the real unit interval, $[0, 1]$, but can have values in a general residuated lattice. I consider this approach very interesting, perspective, and worth attention.

I have the following comments to the scientific contents of the thesis, which **I wish to discuss at the defence**:

1. P. 2: "*When a denial needs to be processed by FCA, one can easily introduce a negative attribute.*" Does this decrease the efficiency of algorithms?
2. P. 3: "*the law of double negation does not generally hold true in the graded setting.*" However, we could choose a fuzzy negation which satisfies it.
3. The requirement $a^{**} = a$ in (1) drastically reduces the choice on linearly ordered sets of truth values (to interior operators).

As a habilitation should also prove an educational experience, I comment on his skills in presentation and formulations, as can be deduced from the submitted thesis. Except for the first item, they do not require a response at the defence unless the author wants to comment on them.

- Pp. 11–12. This is a weak part of the thesis. Eight occurrences of A should be B . Symbols $\cap, \cup$ denote what is f and g in the rest. Theorem 2 speaks of a , which should be g , and uses the symbol $\downarrow$ which does not belong here.
- P. 2: “*one-valued data*”. I would call it two-valued; a single value does carry any information.
- Formulas at the top of p. 9 are repeated from the preceding paragraph.
- P. 16: Notation $\{\alpha,^{0.5}/\beta\}$ as an abbreviation of $\{^1/\alpha,^{0.5}/\beta\}$ is not introduced and $\{^{0.5}/\beta, \gamma\}$ uses a variable γ by mistake; “at most in degree is can be found” – the word “half” is missing and the sentence is wrongly constructed.
- The author violates the golden rule: “Don’t start a sentence with a symbol.” [D. Knuth: Mathematical Writing. Page 1, point 2].
- P. 18: “*respecively*”.
- P. 21: “* *being identity and globalization*” – I think that “or” is the right connective.
- P. 23: Non-existing references (40)–(42).
- P. 25: The meaning of operators $^*, \square$ here is unexplained.
- There are errors in the use of articles and singulars/plurals.
- P. 32: Surprisingly, I have found an error in an accepted paper: (iv) $B^{\uparrow\downarrow}$ should be $B^{\downarrow\uparrow}$.
- Technical curiosity: Trying to copy text from the title page of the pdf file, I extracted the title page of the habilitation thesis JAROŠ Jirí: High Performance Computing in Ultrasound Cancer Treatment, Brno, 2017, overwritten by the current contents. You should take more care of document management (at least at the Faculty of Information Technology).

These comments do not reduce the undoubtful scientific contribution of the thesis.

In summary, I acknowledge that **Jan Konečný, Ph.D., proved his academic qualification and he fulfils the required criteria for receiving the degree “Docent”**.

Prague, February 8, 2019

Prof. Ing. Mirko Navara, DrSc.