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On Improving TLS Identification Results Using Nuisance Variables with Application on PMSM

Dominik Friml *CEITEC*

Brno University of Technology
Brno, Czech Republic
Dominik.Friml@ceitec.vutbr.cz

Michal Kozubík

CEITEC

Brno University of Technology
Brno, Czech Republic
Michal.Kozubik@ceitec.vutbr.cz

Pavel Václavek
CEITEC

Brno University of Technology
Brno, Czech Republic
Pavel.Vaclavek@ceitec.vutbr.cz

Abstract—This article presents a novel total least-squares based method for errors-in-variables model identification with a known structure. This method considers the errors of both input and output variables and thus achieves more accurate estimates compared to conventional ordinary least-squares based methods.

The introduced method consists of two recursive total least-squares algorithms connected in a hierarchical structure, which allows for exploitation of nuisance variables and a priori known structure of the identified model. The total least-squares (TLS) method is introduced, and a new "nuisance improved hierarchical total least-squares" (nHTLS) method is derived. Its properties are discussed and proved by simulations. Furthermore, the method is applied in a practical experiment consisting of the state-space identification of the permanent magnet synchronous motor (PMSM). The introduced method is compared with TLS and proven to provide measurably superior dynamical behavior and smaller estimation error of results.

Index Terms—Total Least-Squares, Errors-in-Variables, Hierarchical Total Least-Squares, Nuisance Variables, PMSM Identification

I. INTRODUCTION

Linear model identification is a crucial ingredient for many applications. Noisy inputs are an extensive issue complicating the identification of the linear models, leading to errors-invariables (EIV) regression problems.

While EIV identification of not only linear systems [1] is being frequently solved by total least-squares methods, it still has significant drawbacks. Since the structure of the identified system is often a part of preliminary information, structured total least-squares is used in [2], [3]. Although this method accounts for the structure of the problem, real-time applications often require recursive identification to provide an adaptive solution to the problem or to minimize memory requirements for computations. While there are some optimized algorithms for structured TLS [4] et al., there are no recursive algorithms. The motivation of this paper is to derive a recursive algorithm benefiting from the inclusion of the structure of the EIV problem.

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This paper extends the total least-squares (TLS) algorithm originated by Golub and Van Loan [5], [6] by exploiting weak estimation properties of singular value decomposition through a structure of the given problem. The exploitation benefits from the hierarchical structure, as used for example in [7], [8]. Such methods were also practically implemented in [9], [10] and [11], but they are all based on ordinary least-squares method, unsuitable for errors-in-variables models.

The introduced method is suitable for EIV models, which is demonstrated on state-space permanent magnet synchronous motor (PMSM) identification. Identification of such motor always relies on measured values of currents and rotor mechanical angular speed, which is always affected by a substantial amount of noise, and therefore ordinary least-squares methods should be used only with caution. The identified model can then be adopted by modern control algorithms, as introduced in [12] etc.

Throughout the paper we will use boldfaced uppercase letters for matrices and boldfaced lowercase letters for vectors. All vectors are a column vectors. Nomenclature $\| \bullet \|_F$ means Frobenius norm, while $\| \bullet \|_2$ means ℓ_2 norm. \mathbf{I}_n and $\mathbf{0}_n$ means $n \times n$ identity matrix and zero matrix respectively. Transpose is depicted by \bullet^T , diagonal matrix by diag(\bullet) and rank of matrix by rank(\bullet). Determinant is $| \bullet |$ and operator stacking one columns of a matrix underneath the previous one is $\operatorname{vec}(\bullet)$. Operators $\operatorname{Cov}[\bullet, \bullet]$ and $\mathbb{E}[\bullet]$ denotes covariance and expected value respectively.

In this paper, the following accents are used to depict the basic properties of variables. The meaning of the accents $\overline{\bullet}$, $\widetilde{\bullet}$ and $\widehat{\bullet}$ is noiseless variable, noisy variable, and estimate of the noiseless variable, respectively. Combination of accents $\widehat{\bullet}$ denotes variable that consists of estimates and noisy variables.

II. STATEMENT OF THE PROBLEM

Consider functions $\overline{y}(t)$ and $\overline{h}(t)$ which are linearly related in the form

$$\overline{y}(t) = \overline{\mathbf{h}}(t)^T \theta, \tag{1}$$

however, instead of observing $\overline{y}(t)$ and $\overline{\mathbf{h}}(t)$, we observe

$$\widetilde{h}_i(t) = \overline{h}_i(t) + \delta_{h_i}(t), \quad i = 1, 2, 3, \dots, n$$
 (2)

$$\widetilde{y}(t) = \overline{y}(t) + \delta_y = \overline{\mathbf{h}}(t)^T \theta + \delta_y$$
 (3)

where δ_h and δ_y are mutually uncorrelated normal distributed stochastic signals with zero mean and unknown variance r_h , r_u ,

$$\mathbb{E}[\delta_u] = \mathbb{E}[\delta_{h_i}] = 0 \tag{4}$$

$$Cov[\delta_{h_i}, \, \delta_{h_j}] = 0, \quad i \neq j \tag{5}$$

$$Cov[\delta_{h_i}, \, \delta_y] = 0. \tag{6}$$

Regressor $\mathbf{h}(t)$ has following structure

$$\mathbf{h}(t) = [u(t-n), \ u(t-n-1), \ \dots, \ u(t-1), -y(t-m), \ -y(t-m-1), \ \dots, \ -y(t-1)]^T,$$
 (7)

where the accent of **h** has been omitted, since this structure is the same for all combinations of them.

The problem under study here is the estimation of the parameter θ from measurements $\widetilde{u}(t_0), \ \widetilde{y}(t_0), \ldots, \ \widetilde{u}(T), \ \widetilde{y}(T),$ while following assumptions holds true. The dynamic system is stable and its structure of is known. Number of identified parameters k and structure of \mathbf{h} are therefore unambiguously declared. Also, all the system modes are observable and controllable. Finally, the input signal $\overline{u}(t)$ provides persistent excitation.

III. TOTAL LEAST-SQUARES

Model from (2)-(3) can be rearranged to

$$0 \approx \left[\widetilde{\mathbf{h}}(t)^T, \ \widetilde{y}(t) \right] \left[\theta^T, \ -1 \right]^T = \widetilde{\varphi}(t)^T \vartheta. \tag{8}$$

System of all equations based on (8) for $t \in < t_0, T >$ can be then formulated as

$$\mathbf{0} = \left(\widetilde{\Phi} + [\Delta \mathbf{H}, \, \Delta \mathbf{y}]\right) \vartheta, \quad \widetilde{\Phi} = \left[\widetilde{\mathbf{H}}, \, \widetilde{\mathbf{y}}\right], \tag{9}$$

where ${\bf 0}$ is zero vector or matrix of corresponding size. Note, that corrections $\Delta {\bf H}$ and $\Delta {\bf y}$ had to be added in the equation to ensure, that rank $\left(\widetilde{\Phi} + [\Delta {\bf H}, \, \Delta {\bf y}]\right) = m+n$. Finding optimal corrections and therefore estimate of θ leads to the following TLS minimization problem.

$$\{\hat{\theta}, \Delta \mathbf{H}, \Delta \mathbf{y}\} := \underset{\theta, \Delta \mathbf{H}, \Delta \mathbf{y}}{\operatorname{minimize}} \|[\Delta \mathbf{H}, \Delta \mathbf{y}]\|_{F}$$
subject to $\mathbf{0} = (\widetilde{\Phi} + [\Delta \mathbf{H}, \Delta \mathbf{y}]) \vartheta$ (10)

The existence and uniqueness of the solution of the classical TLS is given by the following theorem proved in [13]:

Theorem 1. Solution of the total least-squares problem. Let

$$\begin{bmatrix} m+n & 1 \\ \widetilde{\mathbf{H}}, & \widetilde{\mathbf{y}} \end{bmatrix} = \mathbf{U} \Sigma \mathbf{V}^T, \text{ where } \Sigma = diag(\sigma_1, \dots, \sigma_{n+m+1})$$

be a singular value decomposition of $\widetilde{\Phi}$ and $\sigma_1 \geq \cdots \geq \sigma_{n+m+1}$ be the singular values of $\widetilde{\Phi}$. After defining of the partitioning

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{v}_{12} \\ \mathbf{v}_{21} & v_{22} \end{bmatrix}, \ \Sigma = \begin{bmatrix} m+n & 1 \\ \Sigma_1 & \mathbf{0} \\ \mathbf{0} & \sigma_{n+m+1} \end{bmatrix},$$

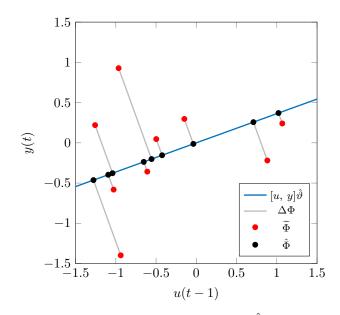


Fig. 1: Visualization of the TLS solution $\hat{\vartheta}$, correction matrix $\Delta\Phi$ and nuisance variables $\hat{\Phi}$ on given data $\widetilde{\Phi}$.

a TLS solution exists if and only if v_{22} is non-zero. In addition, it is unique if and only if $\sigma_{n+m} \neq \sigma_{n+m+1}$. In the case when the TLS solution exists and is unique, it is given by

$$\hat{\theta} = \frac{-\mathbf{v}_{12}}{v_{22}},$$

the corresponding TLS correction matrix is

$$[\Delta \mathbf{H}, \Delta \mathbf{y}] = -\mathbf{U} \operatorname{diag}(\mathbf{0}, \sigma_{n+m+1}) \mathbf{V}^{T}$$

and nuisance variables are

$$\hat{\Phi} = \widetilde{\Phi} + [\Delta \mathbf{H}, \ \Delta \mathbf{y}] = \mathbf{U} \operatorname{diag}(\Sigma_1, 0) \mathbf{V}^T.$$

The TLS is proved [14] to be an unbiased maximum likelihood estimator of the parameters θ , which leads to the conclusion, that for $t \to \infty$, $\hat{\theta} \to \theta$. The conclusion is proved in [5].

The Theorem 1 defined nuisance variables $\hat{\Phi} = \begin{bmatrix} \hat{\mathbf{H}}, \hat{\mathbf{y}} \end{bmatrix}$ as an estimate of the true values $[\overline{\mathbf{H}}, \overline{\mathbf{y}}]$ in such way, that if $\hat{\theta} \to \theta$, than $\hat{\Phi} \to \overline{\Phi}$. The TLS results can be visualized for special case, when $\widetilde{\varphi}(t)^T = [\widetilde{u}(t-1), \widetilde{y}(t)]^T$. Noisy variable $\widetilde{\Phi}$, depicted in the Fig. 1 as black dots, are analyzed and last principal component is found. It is depicted as blue line constructed as $[u, y]\hat{\theta}$, where u and y are horizontal and vertical axis, respectively. This means, that for $\varphi \in \mathbb{R}^2$, first principal component is also solution of TLS problem. Orthogonal projection of measured data $\widetilde{\varphi}$ to first principal component line is thus nuisance variable $\hat{\varphi}$ (red dots).

Batch TLS solution presented by Theorem 1 is unsuitable for realtime application. Therefore, a recursive total least-squares (RTLS) algorithm needs to be provided. To calculate TLS solution recursively, singular value decomposition is avoided for its high computational complexity. Theorem 1

states, that for calculation of $\hat{\theta}$ there is only required the eigenvector $[\mathbf{v}_{12}, v_{22}]^T$ corresponding to the smallest eigenvalue σ_{n+m+1} .

$$\underset{\hat{\theta}}{\text{minimize}} \frac{\hat{\vartheta}\left(\widetilde{\Phi}^T \widetilde{\Phi}\right) \hat{\vartheta}^T}{\hat{\vartheta}^T \hat{\vartheta}} \tag{11}$$

Optimal solution can therefore be found using algorithm 1, which utilizes inverse iteration of constrained Rayleigh quotient (11) to find smallest eigenvector $[\mathbf{v}_{12}, v_{22}]^T$. Detailed derivation can be found in [15] . Note that Algorithm 1

Algorithm 1 Recursive total least-squares (RTLS)

1: **for**
$$t \leftarrow t_0$$
 to T **do**
2: input: $\hat{\theta}(t-1)$, $\mathbf{P}(t-1)$, $\widetilde{\varphi}(t)$
3: $\mathbf{P}(t) = \mathbf{P}(t-1) - \frac{\mathbf{P}(t-1)\left(\widetilde{\varphi}(t)^T \widetilde{\varphi}(t)\right)\mathbf{P}(t-1)}{1+\widetilde{\varphi}(t)\mathbf{P}(t-1)\widetilde{\varphi}(t)^T}$
4: $\mathbf{v}(t) = \mathbf{P}(t) \left[\hat{\theta}(t-1)^T, -1\right]^T$
5: $\hat{\theta}(t) = \frac{-[v_1(t), v_2(t), \dots, v_{n+m}(t)]^T}{v_{n+m+1}(t)}$
6: $\hat{\varphi}(t) = \widetilde{\varphi}(t) - \frac{\widetilde{\varphi}(t)^T \left[\theta(t)^T, -1\right]^T}{\left[\theta(t)^T, -1\right]\left[\theta(t)^T, -1\right]^T} \left[\theta(t)^T, -1\right]^T$
7: output: $\hat{\theta}(t)$, $\mathbf{P}(t)$, $\hat{\varphi}(t)$
8: **end for**

does not provide required variables for calculation of nuisance variables corresponding to Theorem 1. As apparent from properties of SVD, they are therefore calculated as an orthogonal projection of the measured data $\widetilde{\phi}$ to the hyperplane defined by the variable $\widehat{\theta}$.

IV. PROPOSED ALGORITHM

The proposed algorithm exploits the unused structure of the problem and weak estimation property of the calculated nuisance variables.

Nuisance variables $\hat{\phi}$ and correction matrix $\Delta \phi$ are a byproduct of calculation of the $\hat{\theta}$ using singular value decomposition. In real applications, they are usually discarded or not computed at all to reduce computational load, especially in the case of the RTLS. In such cases, Algorithm 1 is simplified by omitting line 6.

While the nuisance variables are proved by [16] to be weak estimator of the true values $\overline{\phi}$, it provides increasingly better estimates as $\hat{\theta} \to \theta$.

As appreciable from the previous chapter, the TLS method does not account for the structure of the identified system. While the structured total least-squares method could be used to solve this issue, there is no recursive equivalent to the method, and therefore it is not usable for successive improvement of previous identification results. The fact, that the regressor $\widetilde{\varphi}(t)$ mostly consists of delayed data from past

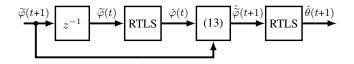


Fig. 2: Visualization of nHTLS algorithm.

regressor $\widetilde{\varphi}(t-1)$ (as apparent from (7)) in combination with the obtained estimate of the past regressor $\hat{\phi}(t+1)$ can be exploited using hierarchical structure of two RTLS blocks, as depicted in the Fig. 2.

Proposed nuisance improved hierarchical total least-squares (nHTLS) algorithm therefore consists of two consequent RTLS blocks, where the first one provides nuisance variables. An nuisance improved regressor for next block $\hat{\widetilde{\varphi}}(t+1)$ is constructed as an combination of the noisy regressor $\widetilde{\varphi}(t+1)$ (7) and last nuisance variables $\hat{\varphi}(t)$ provided by the first RTLS block.

$$\hat{\varphi}(t) = [\hat{u}(t-n), \ \hat{u}(t-n-1), \dots, \ \hat{u}(t-1), \\
-\hat{y}(t-m), \ -\hat{y}(t-m-1), \dots, \ -\hat{y}(t-1), \ -\hat{y}(t)]^T \qquad (12)$$

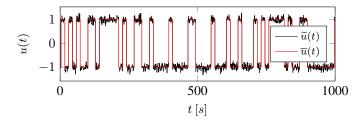
$$\hat{\varphi}(t+1) = [\hat{u}(t-n-1), \ \hat{u}(t-n-2), \dots, \ \widetilde{u}(t), \\
-\hat{y}(t-m-1), \ -\hat{y}(t-m-2), \dots, \ -\hat{y}(t), \ -\widetilde{y}(t+1)]^T \quad (13)$$

Notice, that since nuisance variables $\hat{\varphi}(t)$ lies in the principal subspace of the noisy data $\widetilde{\varphi}(t)$, as follows from properties of the singular decomposition, nuisance improved regressor $\widetilde{\varphi}(t+1)$ also lies in the subspace of the noisy data and therefore existence and unbiasness of the solution $\hat{\theta}_{nHTLS}$ directly follows from the Theorem 1.

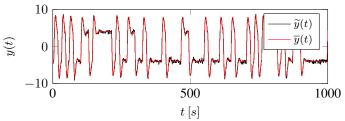
Creating a hierarchical structure consisting of two RTLS algorithms, where one uses regressor $\widetilde{\varphi}(t)$ and the second one $\hat{\varphi}(t+1)$ leads to the superior results for $T \not\to \infty$ compared to plain RTLS.

V. SIMULATION RESULTS

For simulations, arbitrary stable 3rd order system from (1) has been selected, where $\theta = \begin{bmatrix} -0.9, \ 0.4, \ -0.9, \ -0.7, \ 1.6 \end{bmatrix}^T$ and following structure of the regressor $\varphi(t) = \begin{bmatrix} u(t-3), \ u(t-2), \ u(t-1), \ -y(t-2), \ -y(t-1), \ -y(t) \end{bmatrix}^T$. Initial conditions for all simulations are $\hat{\theta}(0) = \begin{bmatrix} 1, \ 0, \ 0, \ 0, \end{bmatrix}^T$ and $\mathbf{P}(0) = \mathbf{I}_6 10^6$. Measurement noise is simulated by adding $\delta_h \approx \mathcal{N}(0, \sigma_h)$ and $\delta_u \approx \mathcal{N}(0, \sigma_y)$. Sample of the input

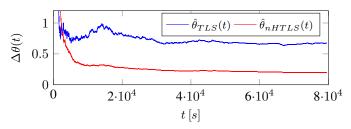


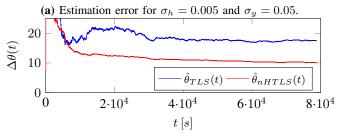
(a) Sample of the true $\overline{u}(t)$ and noisy $\widetilde{u}(t)$ input data.



(b) Sample of the true $\overline{y}(t)$ and noisy $\widetilde{y}(t)$ output data.

Fig. 3: Sample of the data, where $\sigma_h = 0.01$ and $\sigma_y = 0.1$.





(b) Estimation error for $\sigma_h = 0.01$ and $\sigma_y = 0.1$.

Fig. 4: Estimation error of simulated scenarios.

u(t) and output y(t) data with no noise and with simulated measurement noise is presented in the figure 3.

Since true values of θ are a priori known, effectivity of nRTLS can be observed on Euclidean distance of the estimate from true value in time

$$\Delta \theta(t) = \|\theta - \hat{\theta}(t)\|_2^2. \tag{14}$$

Simulation with no additional noise $\delta_h=0$ and $\delta_y=0$ confirms the unbiasedness of the nRTLS, since the estimation error tends to zero for both methods.

Euclidean distance of the simulations with nonzero noise is presented in the Fig. 4. Results shows, that $\hat{\theta}_{nHTLS}$ provides better results compared to $\hat{\theta}_{TLS}$ irrespective of number of samples N.

VI. APPLICATION ON IDENTIFICATION OF PMSM

The introduced nHTLS method is, among other models, usable for the identification of the state-space model of permanent magnet synchronous motors (PMSM) in dq reference frame. State-space model of PMSM

$$\begin{split} i_d(t+1) &= \left(1 - \frac{T_s R_s}{L_d}\right) i_d(t) + \frac{T_s p_p L_q}{L_d} \omega_m(t) i_q(t) + \\ &+ \frac{T_s}{L_d} u_d(t), \end{split} \tag{15}$$

$$i_{q}(t+1) = \left(1 - \frac{T_{s}R_{s}}{L_{q}}\right)i_{q}(t) - \frac{T_{s}p_{p}\Psi_{PM}}{L_{q}}\omega_{m}(t) + \frac{-T_{s}p_{p}L_{d}}{L_{a}}\omega_{m}(t)i_{d}(t) + \frac{T_{s}}{L_{a}}u_{q}(t),$$
(16)

$$\omega_{m}(t+1) = \omega_{m}(t) + \frac{3T_{s}p_{p}\Psi_{PM}}{2J}i_{q}(t) + + \frac{3T_{s}p_{p}\left(L_{d}-L_{q}\right)}{2J}i_{d}(t)i_{q}(t), \tag{17}$$

is not linear. The only kind of nonlinearity present is the multiplication of two state variables as presented in (18), (19).

$$\begin{bmatrix} i_{d}(t+1) \\ i_{q}(t+1) \\ \omega_{m}(t+1) \end{bmatrix} = \mathbf{A} \begin{bmatrix} i_{d}(t) \\ i_{q}(t) \\ \omega_{m}(t) \end{bmatrix} + \mathbf{F} \begin{bmatrix} \omega_{m}(t)i_{d}(t) \\ \omega_{m}(t)i_{q}(t) \\ i_{d}(t)i_{q}(t) \end{bmatrix} + \mathbf{B} \begin{bmatrix} u_{d}(t) \\ u_{q}(t) \end{bmatrix}$$
(18)

$$\mathbf{A} = \begin{bmatrix} 1 - \frac{T_s R_s}{L_d} & 0 & 0\\ 0 & 1 - \frac{T_s R_s}{L_q} & -\frac{T_s p_p \Psi_{PM}}{L_q}\\ 0 & \frac{3T_s p_p \Psi_{PM}}{2J} & 1 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 0 & \frac{T_s p_p L_q}{L_d} & 0\\ -\frac{T_s p_p L_d}{L_q} & 0 & 0\\ 0 & 0 & \frac{3T_s p_p (L_d - L_q)}{2J} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \frac{T_s}{L_d} & 0\\ 0 & \frac{T_s}{L_q}\\ 0 & 0 \end{bmatrix}$$
(19)

where i_d and i_q are stator voltage components in dq frame respectively and ω_m is rotor mechanical angular speed. Parameters R_s , L_d , L_q , Ψ_{PM} and J are stator winding resistance, rotor inductance component of d axis, rotor inductance component of q axis, permanent magnet flux and moment of inertia respectively and constants p_p and T_s are number of pole pairs and sample period of measured data.

For the purposes of the identification, the problem can be linearized by replacing the vector of multiplied state variables $\left[\omega_m(t)i_d(t) \quad \omega_m(t)i_q(t) \quad i_d(t)i_q(t)\right]^T$ by fabricated exogenous signal vector calculated by (20). Parameters for such fabricated signals are presented in model 18 as **F**. This results in linear model with no approximations and therefore there is no drawback in using such linearized model.

$$\begin{bmatrix} \omega_m i_d(t) \\ \omega_m i_q(t) \\ i_d i_q(t) \end{bmatrix} = \begin{bmatrix} \omega_m(t) \\ \omega_m(t) \\ i_d(t) \end{bmatrix} \begin{bmatrix} i_d(t) \\ i_q(t) \\ i_q(t) \end{bmatrix}$$
(20)

Each of the three state equation of the linear model is first order system difference equation with multiple inputs similar to the one in (1). Since the structure is also known, equation can be simplified such that

$$\theta_{1} = [a_{11}, b_{11}, f_{12}]^{T},$$

$$\theta_{2} = [a_{22}, a_{23}, b_{22}, f_{21}]^{T},$$

$$\theta_{3} = [a_{33}, a_{32}, f_{33}]^{T},$$

$$\varphi_{1}(t) = [i_{d}(t-1), u_{d}(t-1), \omega_{m}i_{q}(t-1), i_{d}(t)]^{T},$$

$$\varphi_{2}(t) = [i_{q}(t-1), \omega_{m}(t-1), u_{q}(t-1), \omega_{m}i_{d}(t-1), i_{q}(t)]^{T},$$

$$\varphi_{3}(t) = [\omega_{m}(t-1), i_{q}(t-1), i_{d}i_{q}(t-1), \omega(t)]^{T}.$$
(22)

The RTLS algorithm provides nuisance variables $\hat{\varphi}_1(t)$, $\hat{\varphi}_2(t)$ and $\hat{\varphi}_3(t)$, from which can be constructed nuisance improved regressors

$$\hat{\widetilde{\varphi}}_{1}(t) = \left[\hat{i}_{d}(t-1), \ \widetilde{u}_{d}(t-1), \ \hat{\omega}_{m}\hat{i}_{q}(t-1), \ \widetilde{i}_{d}(t)\right]^{T},$$

$$\hat{\widetilde{\varphi}}_{2}(t) = \left[\hat{i}_{q}(t-1), \ \hat{\omega}_{m}(t-1), \ \widetilde{u}_{q}(t-1), \ \hat{\omega}_{m}\hat{i}_{d}(t-1), \ \widetilde{i}_{q}(t)\right]^{T},$$

$$\hat{\widetilde{\varphi}}_{3}(t) = \left[\hat{\omega}_{m}(t-1), \ \hat{i}_{q}(t-1), \ \hat{i}_{d}\hat{i}_{q}(t-1), \ \widetilde{\omega}_{m}(t)\right]^{T}.$$
(23)

Nuisance improved regressors are inputs to the hierarchical RTLS resulting in a similar structure as presented in Fig. 2 with the only difference that the three algorithms are mutually connected with (23).

VII. RESULTS OF APPLICATION ON PMSM

Identification data are measured on TG Drives TGT3 (see Fig. 5) low voltage PMSM motor with sampling period $62.5\,\mu s$. Stator currents and voltages in abc frame were measured and transformed to dq frame. Rotor position is sensed by the incremental encoder with 4096 edges per mechanical revolution, and ω_m is reconstructed using an EEMF observer. Data are normalized to ± 1 . Details can be studied in [17].

Obtained data are separated into identification and verification sections, each 33 s long.

Input signals u_d and u_q are independent and identically distributed pseudorandom binary sequences providing persistent excitation. Its sample can be observed in Fig. 6.

Initial conditions for all identifications are

$$\hat{\theta}_1(0) = \begin{bmatrix} 0, 1, 1 \end{bmatrix}^T, \hat{\theta}_2(0) = \begin{bmatrix} 0, 1, 1, 1 \end{bmatrix}^T, \hat{\theta}_3(0) = \begin{bmatrix} 0, 1, 1 \end{bmatrix}^T,$$

$$\mathbf{P}_1(0) = \mathbf{I}_4 10^6, \ \mathbf{P}_2(0) = \mathbf{I}_5 10^6, \ \mathbf{P}_3(0) = \mathbf{I}_4 10^6.$$
(24)

Regressor noise is yielded by measurement and preprocessing of the data. Simulated noise $\delta_h = \delta_y \approx \mathcal{N}(0, 0.01)$ is added after normalization for second test.

Since parameters of the model are assumed to be unknown, application of Euclidean distance is not usable in this case. Instead, models defined by $\hat{\theta}(t)$ are compared to the measured data and Akaike information criteria are calculated.

$$AIC = 2k - 2\ln \mathcal{L} \tag{25}$$

Equation (25) defines Akaike information criteria (AIC), where k is the number of estimated parameters and $\ln \mathcal{L}$ is the likelihood of the compared model.



Fig. 5: Laboratory setup for data acquisition.

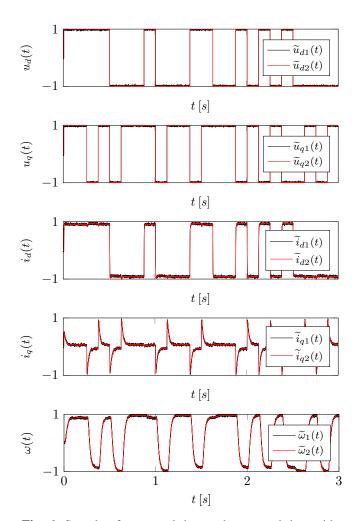


Fig. 6: Sample of measured data and measured data with additional noise denoted by 1 and 2 respectively.

For this likelihood to be defined, the following substitutions have to be stated

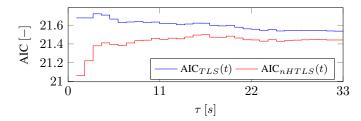
$$\mathbf{x}(t) = \begin{bmatrix} i_d(t) \\ i_q(t) \\ \omega_m(t) \end{bmatrix}, \ \mathbf{f}(t) = \begin{bmatrix} \omega_m(t)i_d(t) \\ \omega_m(t)i_q(t) \\ i_d(t)i_q(t) \end{bmatrix}, \ \mathbf{u}(t) = \begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix},$$

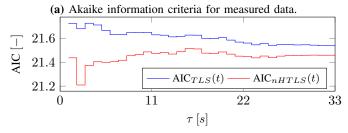
$$\tilde{\mathbf{a}}(t) = \begin{bmatrix} \tilde{\mathbf{x}}(t) \\ \tilde{\mathbf{f}}(t) \\ \tilde{\mathbf{u}}(t) \\ \tilde{\mathbf{x}}(t+1) \end{bmatrix}, \ \hat{\mathbf{a}}(t) = \begin{bmatrix} \hat{\mathbf{x}}(t) \\ \hat{\mathbf{f}}(t) \\ \bar{\mathbf{u}}(t) \\ \left[\hat{\mathbf{A}}, \ \hat{\mathbf{F}}, \ \hat{\mathbf{B}} \right] \left[\hat{\mathbf{x}}(t), \ \hat{\mathbf{f}}(t), \ \bar{\mathbf{u}}(t) \right]^T \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}(t_0)^T \\ \vdots \\ \mathbf{a}(T)^T \end{bmatrix}, \ \mathbf{d} = \text{vec}\left(\tilde{\mathbf{A}}\right) - \text{vec}\left(\hat{\mathbf{A}}\right), \tag{26}$$

where $\hat{\bf A}$, $\hat{\bf F}$ and $\hat{\bf B}$ originate from substituting identified parameters $\hat{\theta}_1(\tau)$, $\hat{\theta}_2(\tau)$ and $\hat{\theta}_3(\tau)$ to (19) using (21), where τ is identification step. Extended regressor $\tilde{\bf a}$ is constructed from measured data, while extended regressor $\hat{\bf a}$ is constructed from simulation data of identified model.

Using substitutions (26), log-likelihood function can be





(b) Akaike information criteria for measured data with added noise.

Fig. 7: Akaike information criteria for both experiments.

defined as

$$\ln \mathcal{L} = -\frac{1}{2} \mathbf{d}^T \mathbf{P} \mathbf{d} - \frac{11}{2} \ln 2\pi - \frac{1}{2} \ln |\mathbf{P}^{-1}|, \qquad (27)$$

where ${\bf P}$ is corresponding precision matrix. For derivation details refer to [18]. Since noise variances r_h , r_u are not known as stated in chapter II, precision matrix can not be determined. Since it is the same for both compared identification results, it can be chosen arbitrarily without prioritization of any of the models. Therefore, it is defined as ${\bf P}={\bf I}_{11}10^6$ for comparison purposes. The resulting information criteria can be observed in the Fig. 7. The lower the criteria, the higher probability of the model to be true. As apparent, nHTLS provides a better model compared to RTLS for any number of samples, which corresponds to the simulation results.

VIII. CONCLUSION

To improve the dynamic identification properties of recursive total least-squares (RTLS) algorithm, a novel recursive identification method is proposed. This method exploits the known structure of the identified system and uses nuisance variables through a hierarchical structure of RTLS blocks. This nuisance improved hierarchical total least-squares (nHTLS) algorithm leads to superior dynamical properties compared to RTLS while retaining unbiased estimate for given conditions, meaning that nHTLS achieves a more probable estimate for the limited number of samples and the same estimate for a theoretically infinite number of samples compared to RTLS.

Proposed algorithm properties are proved by thorough simulations presented in chapter V. Successful application on PMSM state-space identification proved the practical usefulness of the presented algorithm.

While nHTLS is applicable, further algorithm optimizations and enhancements are possible. E.g., minimizing computational complexity, memory requirements, or implementing forgetting capability to allow for adaptability of the algorithm. Such enhancements are the objective of future investigation.

APPENDIX A NHTLS ALGORITHM

Algorithm 2 nHTLS

- 1: **for** $t \leftarrow t_0$ to T **do**
- 2: input: $\hat{\theta}(t-1)$, $\hat{\theta}_1(t-2)$, $\mathbf{P}_1(t-2)$, $\mathbf{P}_2(t-1)$, $\widetilde{\varphi}(t-1)$, $\widetilde{y}(t)$
- 3: $\hat{\theta}_1(t-1), \mathbf{P}_1(t-1), \hat{\varphi}(t-1) = \text{RTLS}\left(\hat{\theta}_1(t-2), \mathbf{P}_1(t-2), \widetilde{\varphi}(t-1)\right)$
- 4: $\hat{\widetilde{\varphi}}(t) = [\hat{u}(t-n-2), \ \hat{u}(t-n-3), \dots, \ \widetilde{u}(t-1), \\ -\hat{y}(t-m-2), \ -\hat{y}(t-m-3), \dots, \ -\hat{y}(t-1), \ -\widetilde{y}(t)]^T$
- 5: $\hat{\theta}(t), \mathbf{P}_2(t) = \text{RTLS}\left(\hat{\theta}(t-1), \mathbf{P}_2(t-1), \hat{\widetilde{\varphi}}(t)\right)$
- 6: output: $\hat{\theta}(t)$, $\mathbf{P}_{2}(t)$, $\hat{\theta}_{1}(t-1)$, $\mathbf{P}_{1}(t-1)$
- 7: end for

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