Halfsemestral exam BAYa - 1. 11. 2023 - A,

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1. What are Bayesian networks useful for? What properties of the probabilitstic models can be expressed with them? What process is easy to visualize with Bayesian Networks?

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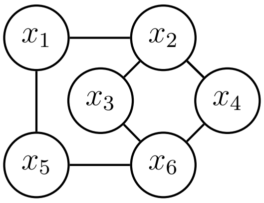
2. Draw the Bayesian network for the following joint probability:

$$p(a, b, c, d) = p(a)p(b|a)p(c)p(d|a, c, b)$$

Now draw also the corresponding factor graph, where there is a factor for each term in the right hand side for this equation. Can belief propagation algorithm be applied in such factor graph? Why?

3. What is, in general, the sum-product message passing algorithm? What can we use it for? What are its limitations? What is the fundamental idea behind the sum-product message passing algorithm that makes it more efficient than the brute-force approach? Do NOT derive the algorithm, answer the questions in general terms.

4. Given the following MRF, say whether the following statements are true or false and explain why.



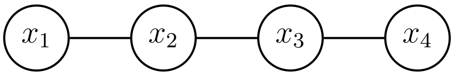
(a) $p(x_1, x_6|x_2, x_5) = p(x_1|x_2, x_5)p(x_6|x_2, x_5)$

(b) $p(x_2, x_6|x_3, x_4) = p(x_2|x_3, x_4)p(x_6|x_3, x_4)$

(c) $p(x_1, x_4|x_3) = p(x_1|x_3)p(x_4|x_3)$

(d) $p(x_1, x_4, x_5|x_2, x_6) = p(x_1|x_2, x_6)p(x_4|x_2, x_6)p(x_6|x_2, x_6)$

5. Given the following chain, estimate the marginal probability of the random variable x_3



To obtain it, first write the joint probability in terms of the potential functions of the Markov random field. Next, express how we can marginalize the corresponding random variable. Finally, reorder the terms for the most efficient estimation of such inference in this chain. Assume that each random variables is discrete (cathegorical) with K cathegories.

6. Draw the Bayesian Network for the Gaussian Mixture Model (GMM) representing the distribution of a single observation x. Write the corresponding factorization for the joint distribution of the observed and hidden random variables x and z. Explain what distributions the individual factors correspond to. From this factorization, derive the equation for evaluating the GMM probability density function (i.e. the marginal probability density $p(x)$).

7. Expectation Maximization (EM) algorithm makes use of the following equality:

$$\ln p(\mathbf{X}|\boldsymbol{\eta}) = \sum_{\mathbf{Z}} q(\mathbf{Z}) \ln p(\mathbf{X}, \mathbf{Z}|\boldsymbol{\eta}) - \sum_{\mathbf{Z}} q(\mathbf{Z}) \ln q(\mathbf{Z}) - \sum_{\mathbf{Z}} q(\mathbf{Z}) \ln \frac{p(\mathbf{Z}|\mathbf{X}, \boldsymbol{\eta})}{q(\mathbf{Z})}$$

How is this equation used in the EM algorithm? What does the term $q(\mathbf{Z})$ represent (in general, not just for GMM training)? How is $q(\mathbf{Z})$ set in the E-step and why? How is $q(\mathbf{Z})$ used in the M-step? Which term from the equation is optimized in the M-step and how?