Halfsemestral exam BAYa - 15. 11. 2024 - A,

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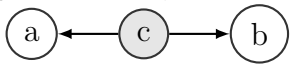
1. Write an equation expressing that variables a and b are conditionally independent given a variable c. Using basic rules of probability (product rule, sum rule, Bayes rule), provide proof that this equation holds for the Bayesian Network: 

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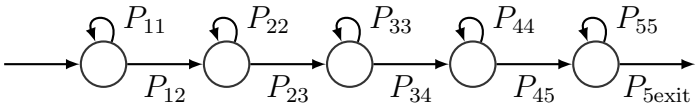
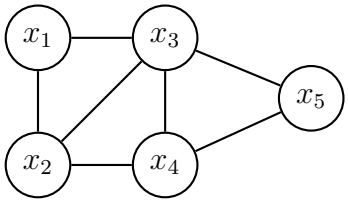
2. Let's have an HMM with 5 states (and their corresponding distributions of observations) and transition probabilities as seen in the picture below. We generate a sequence of 4 observations. Draw the corresponding Bayesian network. What does each node represent in the Bayesian network? Write the corresponding factorization of the joint probability of all the random variables in this model.



3. Given the following MRF, say whether the following statements are true or false and explain why.



(a) $p(x_1, x_5|x_4) = p(x_1|x_4)p(x_5|x_4)$

(b) $p(x_2, x_5|x_4) = p(x_2|x_4)p(x_5|x_4)$

(c) $p(x_1, x_4|x_2, x_3) = p(x_1|x_2, x_3)p(x_4|x_2, x_3)$

(d) $p(x_1, x_2, x_5|x_3, x_4) = p(x_1|x_3, x_4)p(x_2|x_3, x_4)p(x_5|x_3, x_4)$

4. What is brute force marginalization? Give an example.

5. The Maximum Likelihood estimate of the variance parameter of Gaussian distribution is given by formula $\sigma^2_{ML} = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu_{ML})^2$, where x_n are training observations, N is the total number of training observations, and μ_{ML} is the maximum likelihood estimat e of the mean. Derive this formula. What is the objective function that we maximize? How do we maximize it?

6. What is the max-product message-passing algorithm? How is it different from the sum-product algorithm? What problems does it solve? What limitations does it have? What is the backtracking used for in this algorithm?