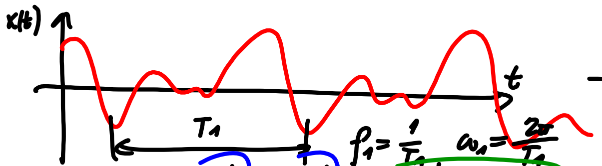
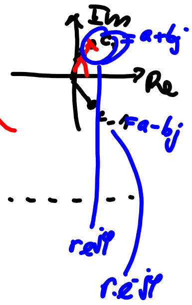




$f_1 = \frac{1}{T_1}$ $\omega_1 = \frac{2\pi}{T_1}$

$x(t) = c_0 + c_1 e^{j\omega_1 t} + c_{-1} e^{-j\omega_1 t} + c_2 e^{j2\omega_1 t} + c_{-2} e^{-j2\omega_1 t} + \dots$

$r e^{j\varphi}$ $r e^{-j\varphi}$

$c_1 = c_{-1}^*$ $c_2 = c_{-2}^*$

$|c_1| = |c_{-1}|$

$\arg c_1 = -\arg c_{-1}$

$C_1 \cos(\omega_1 t + \varphi_1)$ $C_2 \cos(2\omega_1 t + \varphi_2) + \dots$

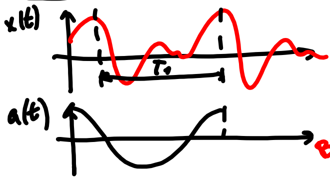
$C_1 = 2|c_1|$ $C_2 = 2|c_2|$

$\varphi_1 = \arg c_1 = -\arg c_{-1}$ $\varphi_2 = \arg c_2 = -\arg c_{-2}$ ...

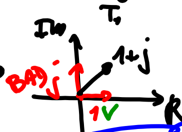
FŘ

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}}$$ FŘ (syntéza)

hledáme c_k!



$c = \int_{T_1} x(t)\, a(t)\, dt$



$\mathrm{Re}(1 \cdot (1+j)) = 1$

$\mathrm{Re}(-j(1+j)) = +1$

$j \cdot -1$

pokud komplexní báze, tak v součinu musí být $a^*(t)$

$c_k = \int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt$

ortogonalita

Báze: $\int_{T_1} e^{jk\omega_1 t} \cdot e^{-jl\omega_1 t}\, dt = \emptyset$ $k \neq l$

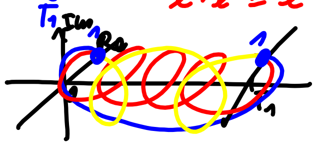
$e^a \cdot e^b = e^{a+b}$

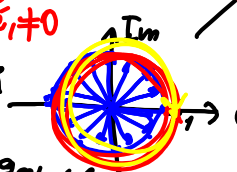
$$\boxed{\int_0^{T_1} e^{j(k-l)\omega_1 t}\, dt = 0}$$

celé, $\neq 0$

|| je nula

$\Rightarrow$ ortogonální





$re^{j\varphi}$

$\int = 0$ $\int = 0$ $|e^{j\text{něco}}| = 1$

velikost báze

$\|e^{jk\omega_1 t}\| = \int_0^{T_1} |e^{jk\omega_1 t}|^2\, dt = \int_0^{T_1} 1^2\, dt = T_1$

$$\boxed{c_k = \frac{1}{T_1} \int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt}$$ FŘ "analyzační"

VÝSTUP = možná nějaká konstanta · Sum. operátor · VSTUP · $e^{\pm j \text{ čas frekvence}}$

frekv → čas
čas → frekv.

FŘ syntéza signálu

$x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{+jtk\omega_1}$

FŘ analýza signálu

$c_k = \frac{1}{T_1} \int_{T_1} x(t)\, e^{-jtk\omega_1}$

DFT analýza

$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-jn\frac{k 2\pi}{N}}$

FŘ příklad 1

$$x(t) = 5\cos(\underbrace{1000\pi}_{\omega_1 = 1000\pi} t + \tfrac{\pi}{4}) =$$

F.Ř. ?

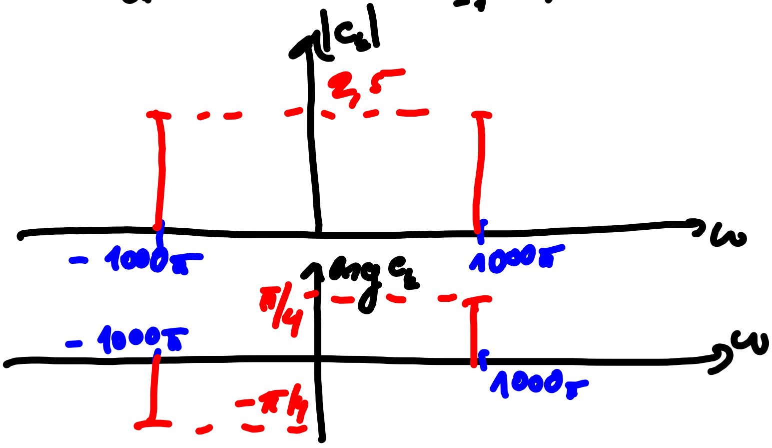
$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$= \frac{5}{2} e^{j(1000\pi t + \frac{\pi}{4})} + \frac{5}{2} e^{-j(1000\pi t + \frac{\pi}{4})} =$$

$$= \underbrace{\frac{5}{2} e^{j\frac{\pi}{4}}}_{c_1} \underbrace{e^{j1000\pi t}}_{e^{j\omega_1 t}} + \underbrace{\frac{5}{2} e^{-j\frac{\pi}{4}}}_{c_{-1}} \underbrace{e^{-j1000\pi t}}_{e^{-j\omega_1 t}}$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}$$

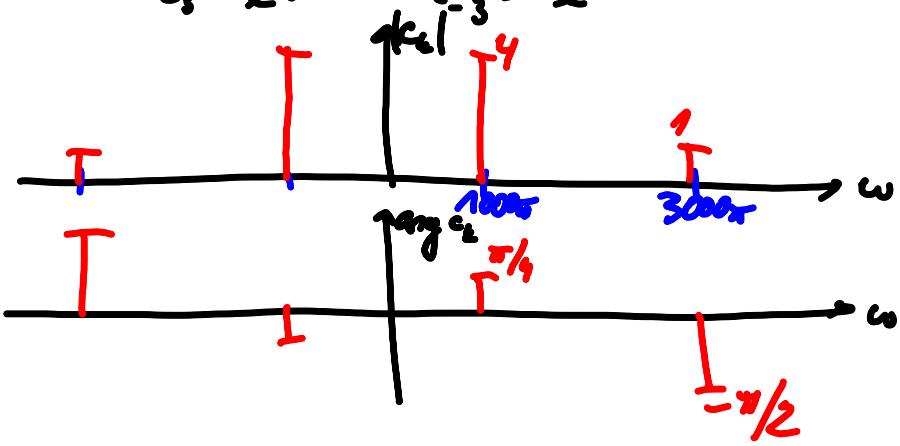
$$c_1 = 2{,}5\, e^{j\frac{\pi}{4}} \qquad c_{-1} = 2{,}5\, e^{-j\frac{\pi}{4}}$$



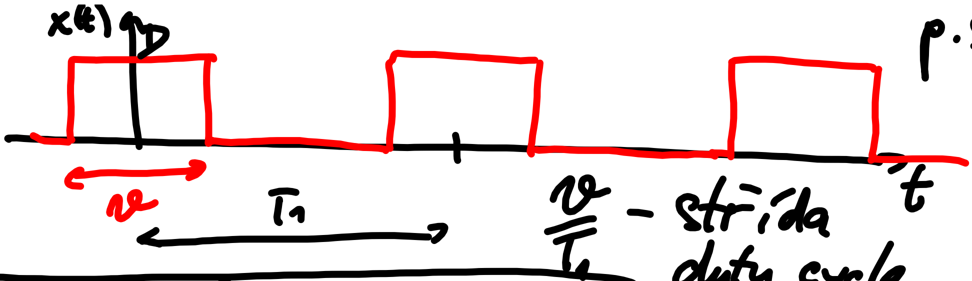
$$x(t) = 8\cos(\underbrace{1000\pi}_{\omega_1} t + \pi/4) + 2\cos(\underbrace{3000\pi}_{3\omega_1} t - \frac{\pi}{2})$$

$c_1 = 4e^{j\pi/4}$ $c_{-1} = 4e^{-j\pi/4}$

$c_3 = e^{-j\pi/2}$ $c_{-3} = e^{+j\pi/2}$



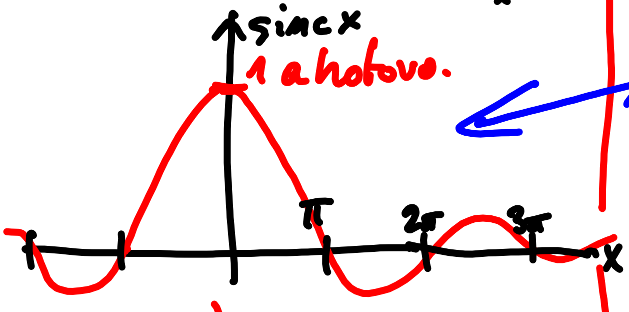
p.s.o.i.



F.R.

$\frac{\vartheta}{T_1}$ – střída
duty cycle

1. rozšíření $\mathrm{sinc}\,x = \frac{\sin x}{x}$



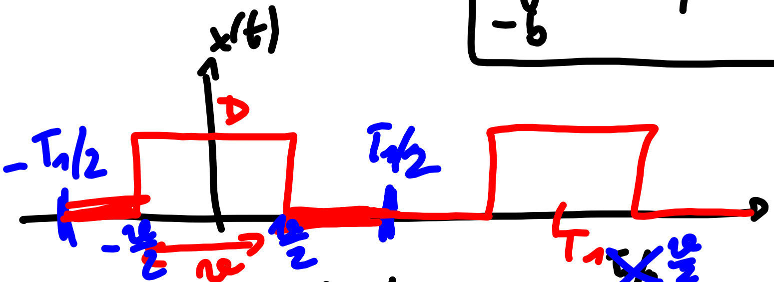
$$\left(\frac{e^{jxy}}{jx}\right)' = \frac{e^{jxy}}{\cancel{jx}}\cdot \cancel{jx} = e^{jxy}$$

Rozšíření č. 2

$$\sin x = \frac{e^{jx}-e^{-jx}}{2j}$$

$$\int_{-b}^{b} e^{\pm jxy}\,dy = \left[\frac{e^{jxy}}{jx}\right]_{-b}^{b} = \frac{e^{jxb}}{jx} - \frac{e^{-jxb}}{jx} = \frac{2}{x}\,\frac{e^{jxb}-e^{-jxb}}{2j} = \frac{2b}{xb}\sin(xb) = 2b\,\mathrm{sinc}(bx)$$

$$\boxed{\int_{-b}^{b} e^{\pm jxy}\,dy = 2b\,\mathrm{sinc}(bx)}$$ S.P.



$$c_k = \frac{1}{T_1}\int_{T_1} x(t)\,e^{-jk\omega_1 t}\,dt = \frac{D}{T_1}\int_{-\cancel{T_1/2}\ -\frac{\vartheta}{2}}^{\cancel{T_1/2}\ \frac{\vartheta}{2}} e^{-jk\omega_1 t}\,dt = \frac{D}{T_1}\int_{-\frac{\vartheta}{2}}^{\frac{\vartheta}{2}} e^{-jk\omega_1 t}\,dt =$$

$$= \frac{D}{T_1}\cdot \cancel{2}\,\cancel{\tfrac{\vartheta}{2}}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right) = \underline{\frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right)}$$