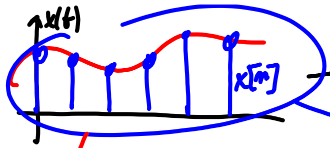
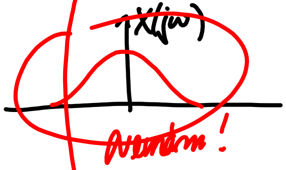
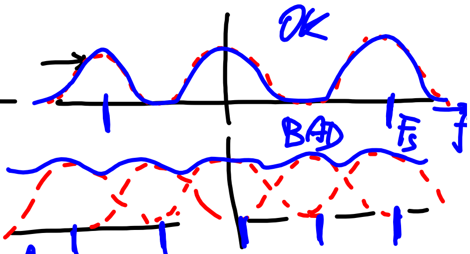
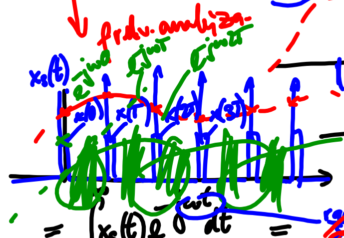


→ spektrální analýza diskrétního signálu ?
→ frekvence







$$X_s(j\omega) = FT\{x_s(t)\} =$$

$$= \int_{-\infty}^{\infty} x_s(t) e^{-j\omega t}\, dt =$$

$$= \sum_{n=-\infty}^{\infty} x(nT) e^{-j\omega nT} =$$

normované kruhová frekvence

$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n \frac{1}{F_s}} =$$

rad/s normovaná ω frekvence.

$$\tilde{X}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$ \tilde{X} (e^{j\omega})

DTFT Discrete-time Fourier transform ← Fourierova transformace s diskrétním časem

≠ DFT

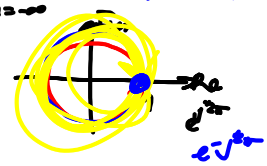
periodické s periodou ?

obyčejná frekvence	–	F_s
kruhová	–"–	$2\pi F_s$
normovaná frekvence		$F_s/F_s = 1$
norm. kruhová frekvence		2π

$e^{a+b} = e^a \cdot e^b$

$$\tilde{X}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j(\omega + k2\pi)n} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \cdot \cancel{e^{-j2\pi kn}} =$$

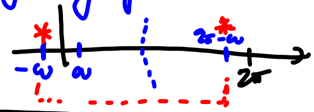
$$= \sum_{n=-\infty}^{\infty} x[n] \boxed{e^{-j\omega n}}$$ periodické se 2π!



$$\tilde{X}(e^{j\omega}) = \tilde{X}^*(e^{-j\omega})$$

$$\tilde{X}(e^{j\omega}) = \tilde{X}^*(e^{j(2\pi-\omega)})$$

moduly stejné argumenty opačné!



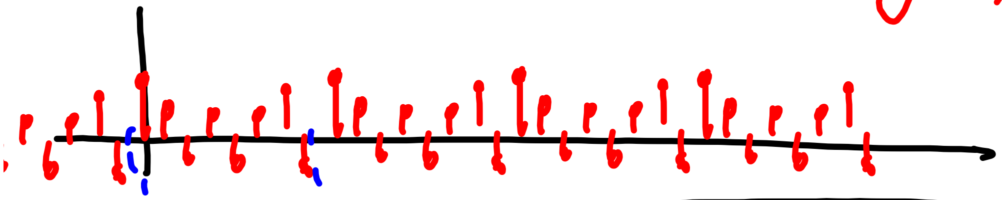
DTFT

1. signál je diskrétní ⇒ DTFT je periodická
2. frekvence je def. všude −∞ do ∞, stačí od 0 do 2π (nebo $-\pi$ do π) nebo od 0 do π, ak $\omega \in \mathbb{R}$!!
3. jedna DTFT – 4 různé frekvenční osy!

zpětná DTFT

$$x[n] = \frac{1}{2\pi} \int^{\pi} \tilde{X}(e^{j\omega}) e^{+j\omega n}$$

DFŘ zpracovává periodické signály



$N = 8$ vzorků

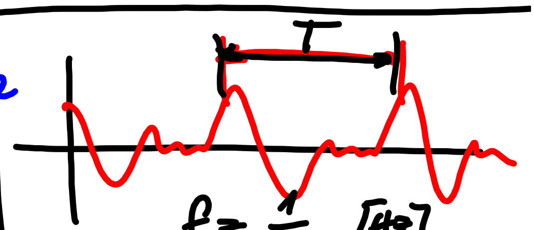
$f_1 = \frac{1}{N}$ [nic]

$\omega_1 = 2\pi f_1 = \frac{2\pi}{N}$

normovaná frekvence

$\omega_1 \to N$???

$N \to \omega_1$ OK.



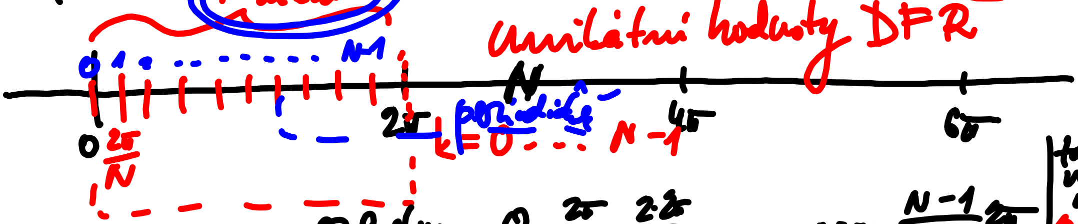
$f = \frac{1}{T}$ [Hz]

obyčejná frekv.

periodisace v čase → diskretisace ve frekvenci

diskretisace v čase → periodisace ve frekvenci

frekvenční osa v normovaných kruh. frekvencích:



umístění hodnoty DFŘ

$k = 0 \dots N-1$

						tady už ne!
norm. ω frekv.:	0	$\frac{2\pi}{N}$	$\frac{2\cdot 2\pi}{N}$	. . . .	$\frac{N-1}{N} 2\pi$	2π
norm. frekv.:	0	$\frac{1}{N}$	$\frac{2}{N}$	. . . .	$\frac{N-1}{N}$	1
frekv. [Hz]:	0	$\frac{F_s}{N}$	$\frac{2F_s}{N}$	. . . . . .	$\frac{N-1}{N} F_s$	F_s
ω frekv. [rad/s]:	0	$\frac{2\pi F_s}{N}$	$\frac{2\cdot 2\pi F_s}{N}$	. . . . .	$\frac{N-1}{N} \cdot 2\pi \cdot F_s$	$2\pi F_s$

k-násobek základní ω frekvence

DFŘ

$$\tilde{X}[k] = \sum_{m=0}^{N-1} \tilde{x}[m] \cdot e^{-jk\frac{2\pi}{N}m}$$

DFS Discrete Fourier Series

IDFŘ

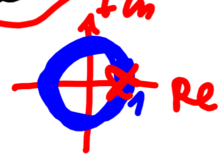
$$\tilde{x}[m] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] e^{+jk\frac{2\pi}{N}m}$$

DFŘ je periodická s N koeficienty ???

$$\tilde{X}[k+N] = \tilde{X}[k]$$

$$e^{a+b} = e^a \cdot e^b$$

$$\boxed{\tilde{X}[k+N]} = \sum_{m=0}^{N-1} \tilde{x}[m] e^{-j(k+N)\frac{2\pi}{N}m} = \sum_{m=0}^{N-1} \tilde{x}[m] e^{-jk\frac{2\pi}{N}m} \cancel{e^{-jN\frac{2\pi}{N}m}} = 1$$



jo!

Analýza DFT cos s periodou N

$$x[n] = C_1 \cos\left(\frac{2\pi}{N} n + \varphi_1\right)$$

chci koeficienty DFT.

$$C_1 \cos(\omega_1 n + \varphi_1)$$

$$\cos\alpha = \frac{e^{j\alpha}+e^{-j\alpha}}{2}$$

$$\cancel{\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{N}kn}}$$

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi}{N}kn}$$

$$-\frac{2\pi}{N}$$

$$x[n] = \frac{C_1}{2} e^{j\left(\frac{2\pi}{N}n+\varphi_1\right)} + \frac{C_1}{2} e^{-j\left(\frac{2\pi}{N}n+\varphi_1\right)} = \frac{C_1}{2} e^{j\varphi_1}\cdot e^{j\frac{2\pi}{N}n} + \frac{C_1}{2} e^{-j\varphi_1}\cdot e^{-j\frac{2\pi}{N}n}$$

$k=1$

$$\tilde{X}[1] = \frac{NC_1}{2} e^{j\varphi_1} \qquad \tilde{X}[N-1] = \frac{NC_1}{2} e^{-j\varphi_1}$$

$$\tilde{X}[k] = \tilde{X}^*[-k] \qquad \tilde{X}[k] = \tilde{X}[k \pm \text{libovolný násobek } N]$$

$$\tilde{X}[-1] = \tilde{X}[N-1] \leftarrow \text{OK!}$$

$$\tilde{X}[k] = \tilde{X}^*[N-k]$$

cosinusovka s amplitudou C_1 a poč. fází φ_1:

$$|\tilde{X}[1]| = |\tilde{X}[N-1]| = \frac{NC_1}{2}$$

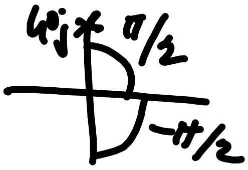
$N=16$

$C_1 = 5$

$\varphi_1 = \frac{\pi}{2}$

$$\arg \tilde{X}[1] = \varphi_1 \qquad \arg \tilde{X}[N-1] = -\varphi_1$$

$40j$ $\qquad -40j$



Diskrétní Fourierova transformace.

$x[n]$ 0 … N-1 → frekvenční analýza

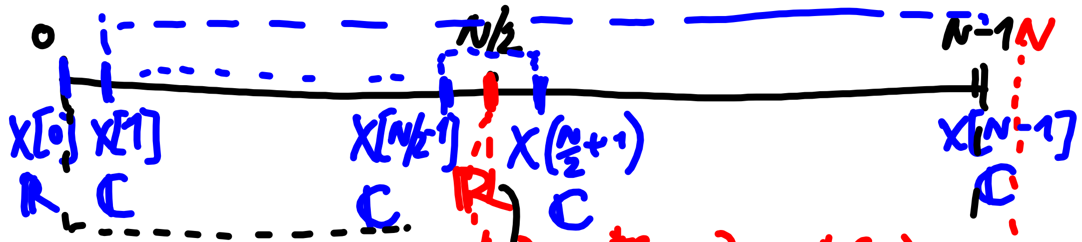
DFT

1. N vzorků v čase → N vzorků ve frekvenci

$$\boxed{X[k]=\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k}{N}n} \quad k \in 0 \ldots N-1}$$

2. $X[k] = X^*[N-k]$

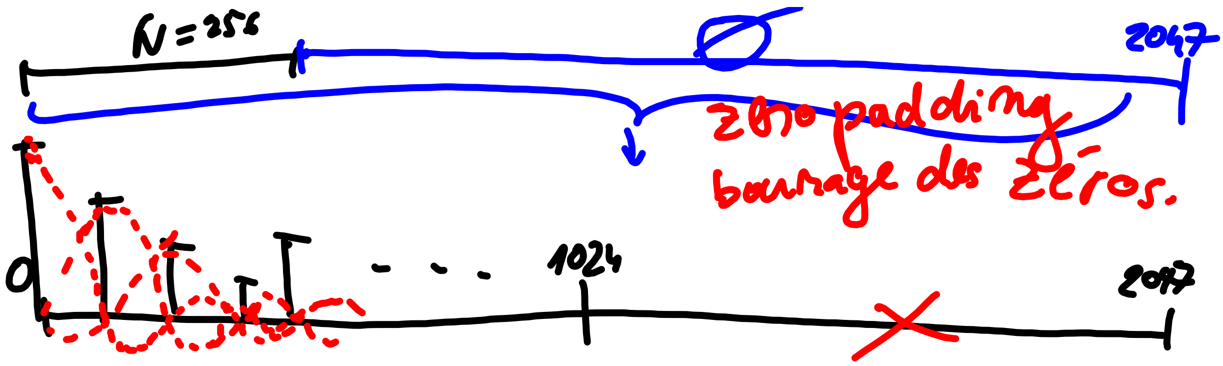
N sudé $X[N/2-1] = X^*[N-(N/2-1)] = X^*[N/2+1]$



$$X\left(\tfrac{N}{2}\right) = X^*\left(N-\tfrac{N}{2}\right) = X^*\left(\tfrac{N}{2}\right)$$

$$1 + 2\left(\tfrac{N}{2}-1\right) + 1 = \cancel{2} + 2\tfrac{N}{2} - \cancel{2} = N$$

norm. frekv.	0	$\frac{1}{N}$ …	$\frac{N-1}{N}$	1
norm. ω	0	$\frac{2\pi}{N}$ …	$2\pi\frac{N-1}{N}$	2π
obyč.	0	$\frac{F_s}{N}$ …	$F_s\frac{N-1}{N}$	F_s
obyč. ω	0	$\frac{2\pi F_s}{N}$ …	$2\pi F_s\frac{N-1}{N}$	$2\pi F_s$



Vlastnosti DFT

$$x[n] \longrightarrow X[k]$$

$$x[\mathrm{mod}_N(n-m)] \longrightarrow X[k]\,e^{-jk\frac{2\pi}{N}m}$$

$$x(t) \longrightarrow X(j\omega)$$

$$x(t-\tau) \longrightarrow X(j\omega)\,e^{-j\omega\tau}$$