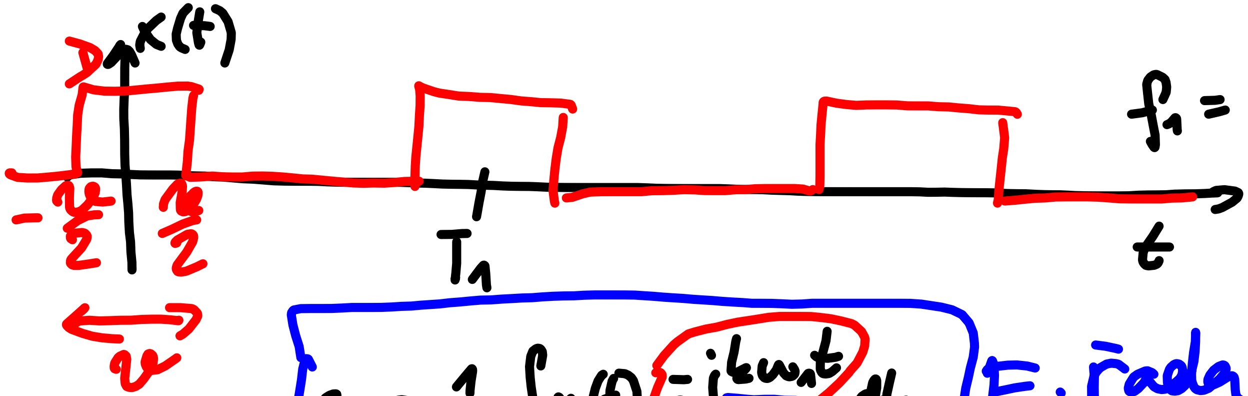


$f_1 = \frac{1}{T_1}$ [Hz]

$\omega_1 = \frac{2\pi}{T_1}$ [rad/s]

$$c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt$$

F. řada analýza

kde? kolik? jak posunuté

$|c_k|$ $\arg c_k$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}$$

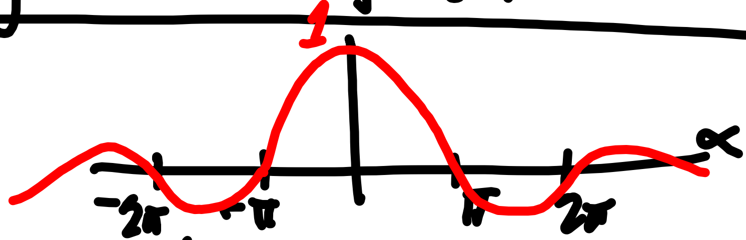
syntéza

čas → frekv −

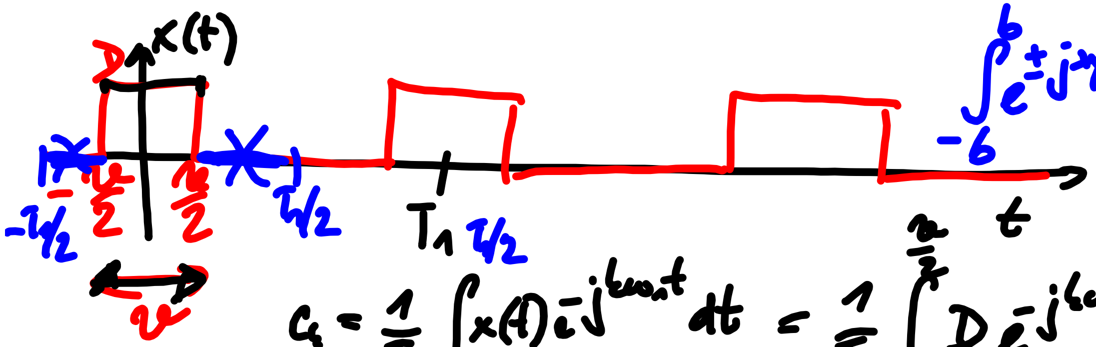
frekv → čas +

Spektrum pro FŘ: polohy a hodnoty koeficientů FŘ.

$\mathrm{sinc}\,\alpha = \frac{\sin \alpha}{\alpha}$



ŠP: $\int_{-b}^{b} e^{\pm jxy}\, dy = 2b\, \mathrm{sinc}(bx)$



$$\int_{-b}^{b} e^{\pm jxy}\,dy = 2b\,\mathrm{sinc}\,bx$$

$y = t$

$x = k\omega_1$

$$c_k = \frac{1}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} x(t)\,e^{-jk\omega_1 t}\,dt = \frac{1}{T_1}\int_{-\frac{\vartheta}{2}}^{\frac{\vartheta}{2}} D\,e^{-jk\omega_1 t}\,dt =$$

$$= \frac{D}{T_1}\,2\cdot\frac{\vartheta}{2}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right) = \frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right)$$

mp.sinc (…/mp.pi)

$5ke^{-}$

$5e^{j0}$

$\frac{\vartheta}{2}\omega = \pi$?

$\omega = \frac{2\pi}{\vartheta}$

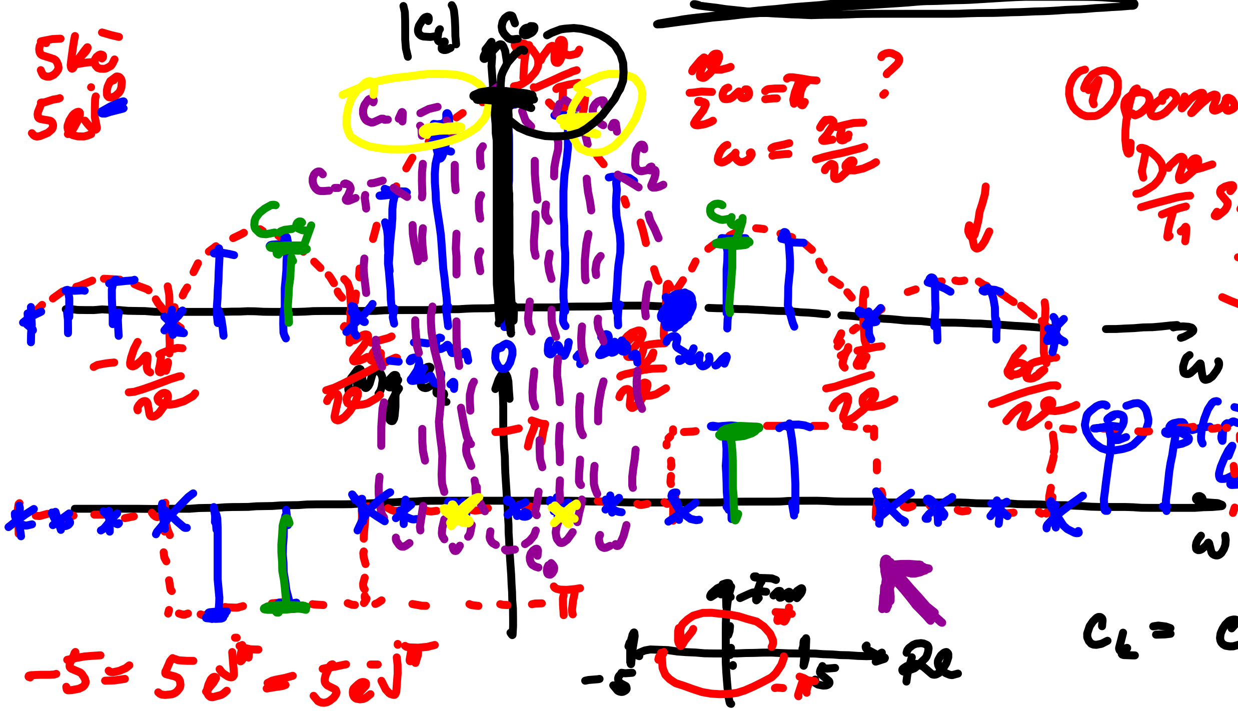
① pomocní funkce

$\frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right)$

→ modul

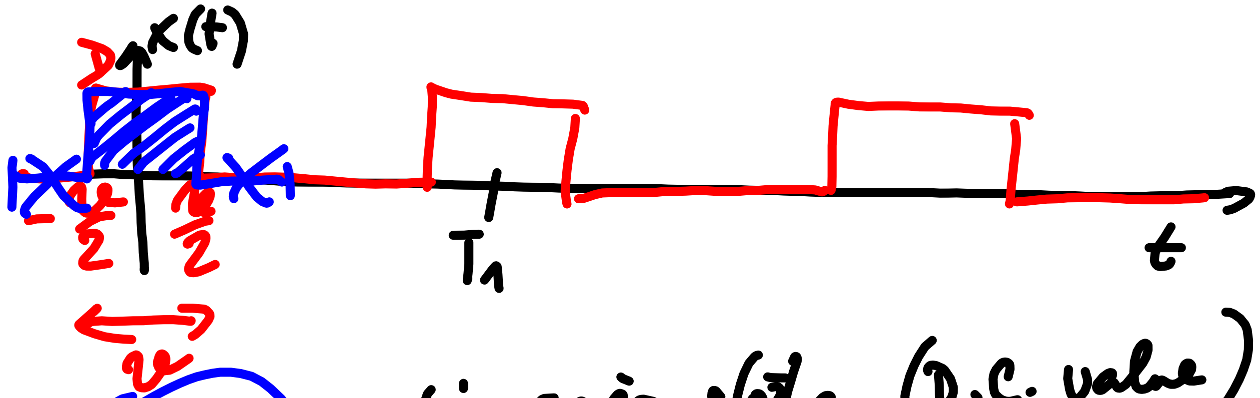
→ argument

② střílet pod ní koeficienty



$-5 = 5e^{j\pi} = 5e^{-j\pi}$

$c_k = c_{-k}^*$ ✓

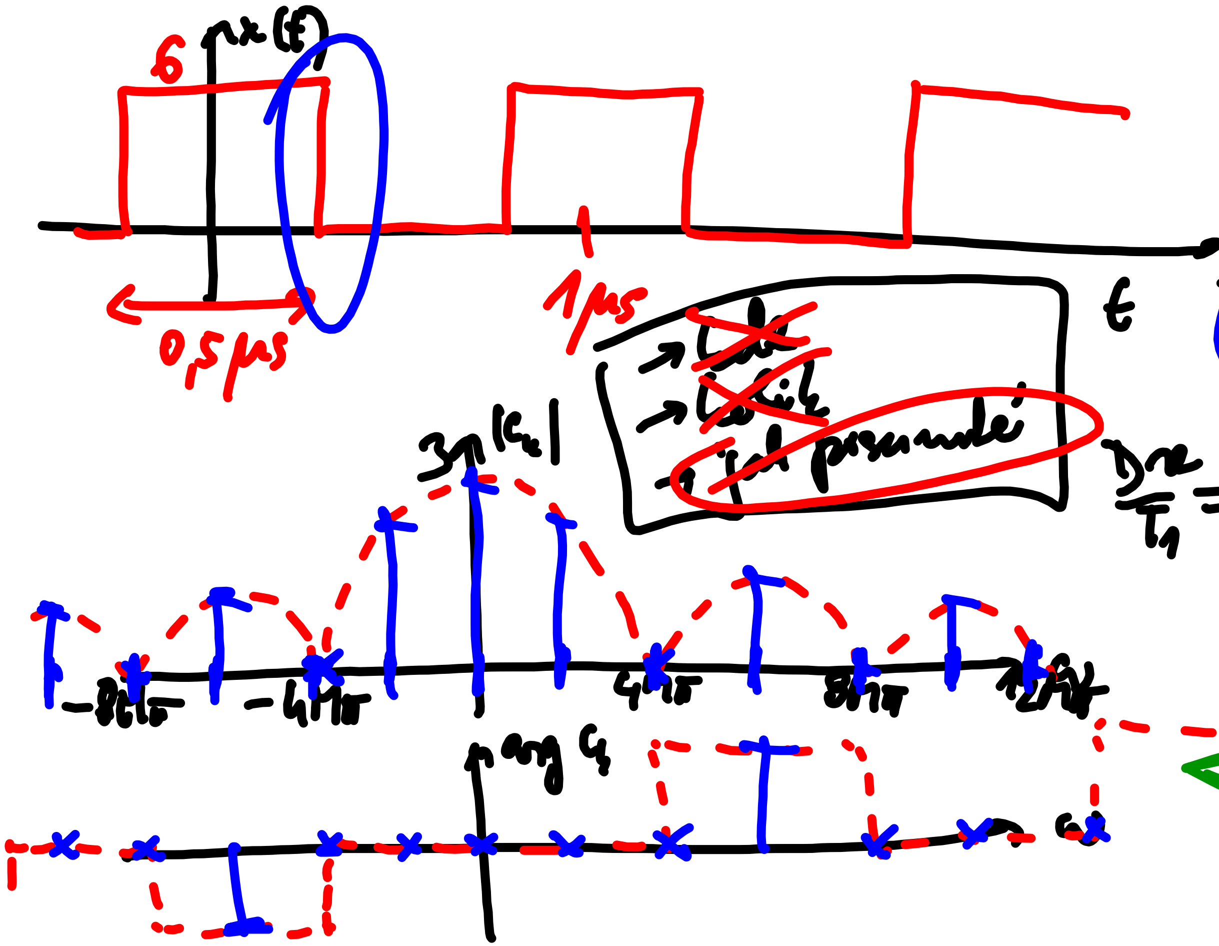


$$c_0 = \frac{D\tau}{T_1}$$

stejnosměr. složka (D.C. value)

$$\bar{x} = \frac{1}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} x(t)\,dt = \frac{1}{T_1}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} D\,dt = \frac{1}{T_1}D\tau$$

✓



$$f_1 = \frac{1}{1\mu s} = 1 MHz$$

$$\omega_1 = 2\cdot 10^6 \pi \; rad/s = 2 M\pi$$

$$\frac{D\tau}{T_1} = 6 \cdot \frac{0{,}5 \cancel{\mu s}}{1 \cancel{\mu s}} = 3$$

$$\frac{2\pi}{\tau} = \frac{2\pi}{0{,}5\cdot 10^{-6}} = 4\pi\cdot 10^6 = 4 M\pi$$

Posunutí signálu a jeho FŘ:

$r = t - \tau$
$t = r + \tau$

$x(t) \longrightarrow c_k$

$x(t-\tau) \longrightarrow \; ? \; c_k' = \frac{1}{T_1}\int_{T_1} x(\underbrace{t-\tau}_{r})\, e^{-jk\omega_1 t}\, dt =$

$= \frac{1}{T_1}\int_{T_1} x(r)\, e^{-jk\omega_1 (r+\tau)}\, dr = \frac{1}{T_1}\int_{T_1} x(r)\, e^{-jk\omega_1 \tau} e^{-jk\omega_1 r}\, dr =$

$e^{a+b} = e^a e^b$

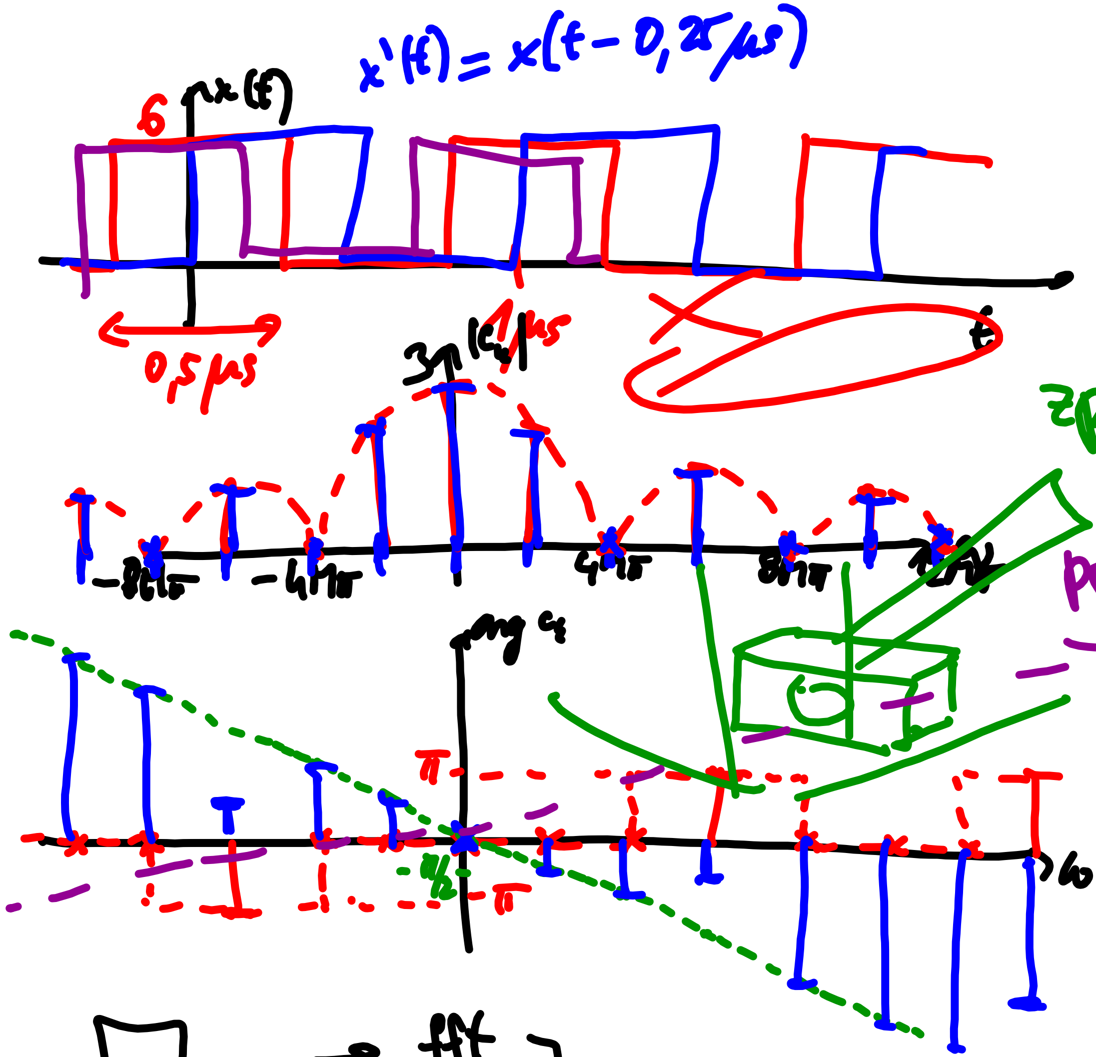
$= e^{-jk\omega_1 \tau} \frac{1}{T_1}\int_{T_1} x(r)\, e^{-jk\omega_1 r}\, dr = \boxed{e^{-jk\omega_1 \tau} \cdot c_k}$

FŘ

Modul: $\boxed{|c_k'|} = |e^{-jk\omega_1\tau} \cdot c_k| = \underbrace{|e^{-jk\omega_1\tau}|}_{1} \cdot |c_k| = \boxed{|c_k|}$ ✓

Arg: $\boxed{\arg c_k'} = \arg e^{-jk\omega_1\tau} + \arg c_k = \boxed{\arg c_k - k\omega_1\tau}$

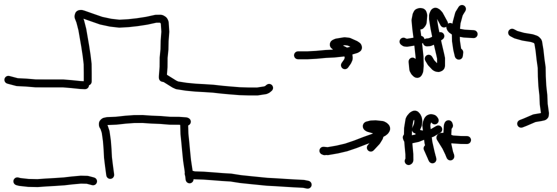
vždy platí



zpoždění ⇒ argument "ujíždí" z kopce

předběhnutí ⇒ do kopce

$\mathrm{ang}\, c_k' = -k\omega_1\tau = \mathrm{ang}\, c_k - k\, 2\cdot \cancel{10^6}\pi \cdot 0{,}25\cdot \cancel{10^{-6}} = \mathrm{ang}\, c_k - k\frac{\pi}{2}$



pozn. num. výp. $\mathrm{ang}\, c_k \in -\pi, \pi$

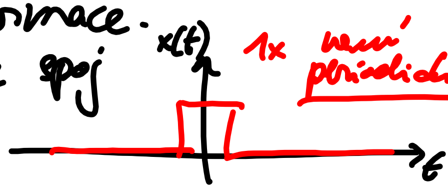
np. unwrap

Fourierova transformace.

F.Ř. neperiodické signály se spoj

$$c_k = \frac{1}{T_1}\int_{-T_1/2}^{T_1/2} x(t)\, e^{-jk\omega_1 t}\, dt$$

$T_1 \to \infty$



1x není periodický

ω_1 neexistuje

$\omega_1 = \frac{2\pi}{T_1} \to d\omega$ "přírůstek kruhové frekv."

$c_k \to dc$ "přírůstek koef. FŘ"

hustota koef FŘ. podle frekv. $\frac{dc}{d\omega}$

hmotnost / objem

$$\frac{\Delta \text{hmotnost}}{\Delta \text{objem}} \to \frac{d\,\text{hmotnost}}{d\,\text{objem}}$$

$$dc = \frac{d\omega}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

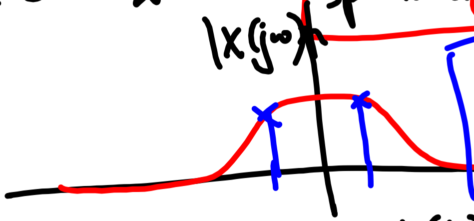
$$T_1 = \frac{2\pi}{d\omega}$$

$$2\pi\frac{dc}{d\omega} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

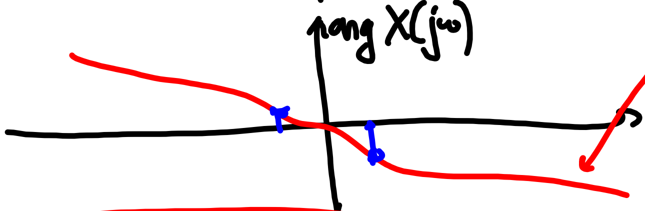
spektrální funkce

F. transformace



$\boxed{X(j\omega)}$ → proč j ???

funkce ω



pro $x(t) \in \mathbb{R}$ platí

$X(j\omega) = X^*(-j\omega)$

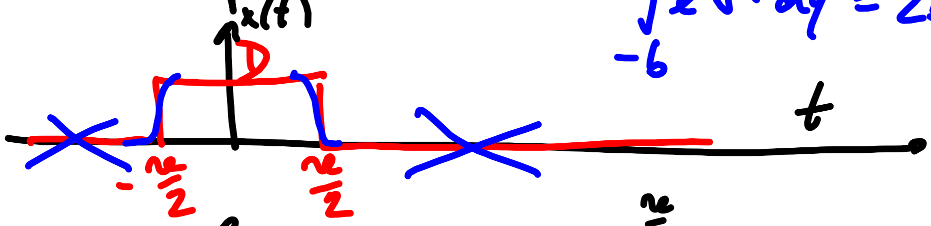
$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega}$$

syntéza

inverzní F.T.

F.T. pro . . . .

$$\int_{-b}^{b} e^{+jxy}\,dy = 2b\,\mathrm{sinc}\,bx \qquad b=\frac{\tau}{2},\; y=t,\; x=\omega$$

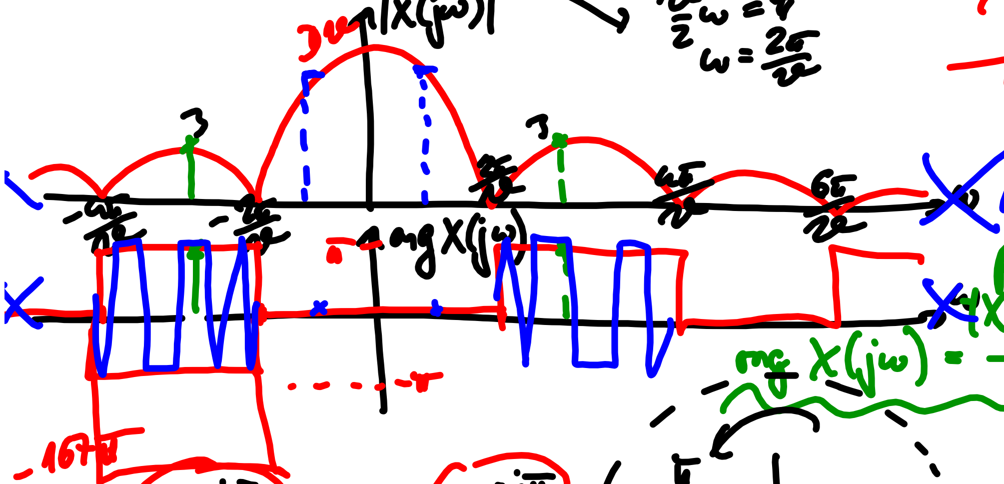


$$X(j\omega)=\int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt = D\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}\,dt = D\,2\cdot\frac{\tau}{2}\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right)=$$

$$= \boxed{D\tau}\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right)$$

$$\frac{\tau}{2}\omega=\pi \qquad \omega=\frac{2\pi}{\tau}$$

spek. funkce



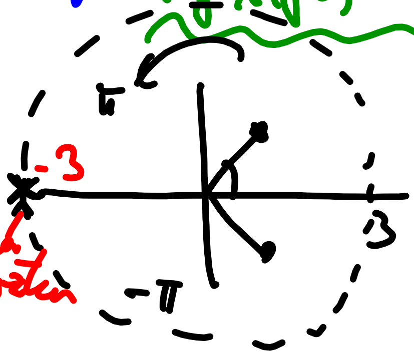
$X(j\omega)=X^*(-j\omega)$

sudá $|X(j\omega)|=|X(-j\omega)|$

lichá $\arg X(j\omega) = -\arg\left(X(-j\omega)\right)$

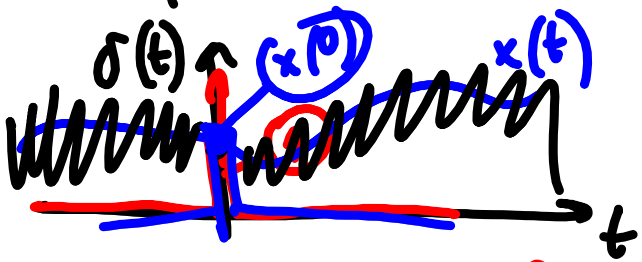
$3e^{j\pi}$ $\quad$ $3e^{-j\pi}$

-3 $\quad$ -3



$-3=(-3)^*$ protože nemá imag. složku

F.T. Diracova impulsu



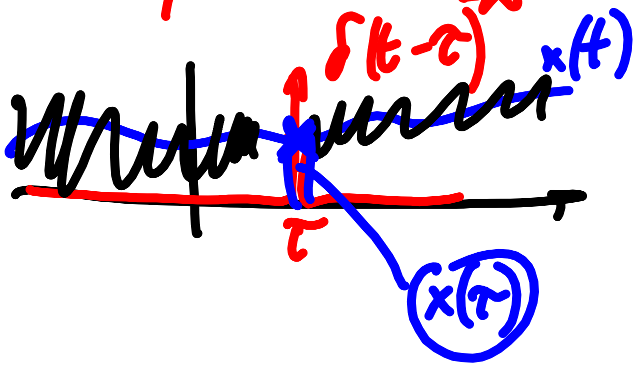
šířka 0

výška ∞

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\,\delta(t)\,dt =$$

$$= \underline{x(0)}$$

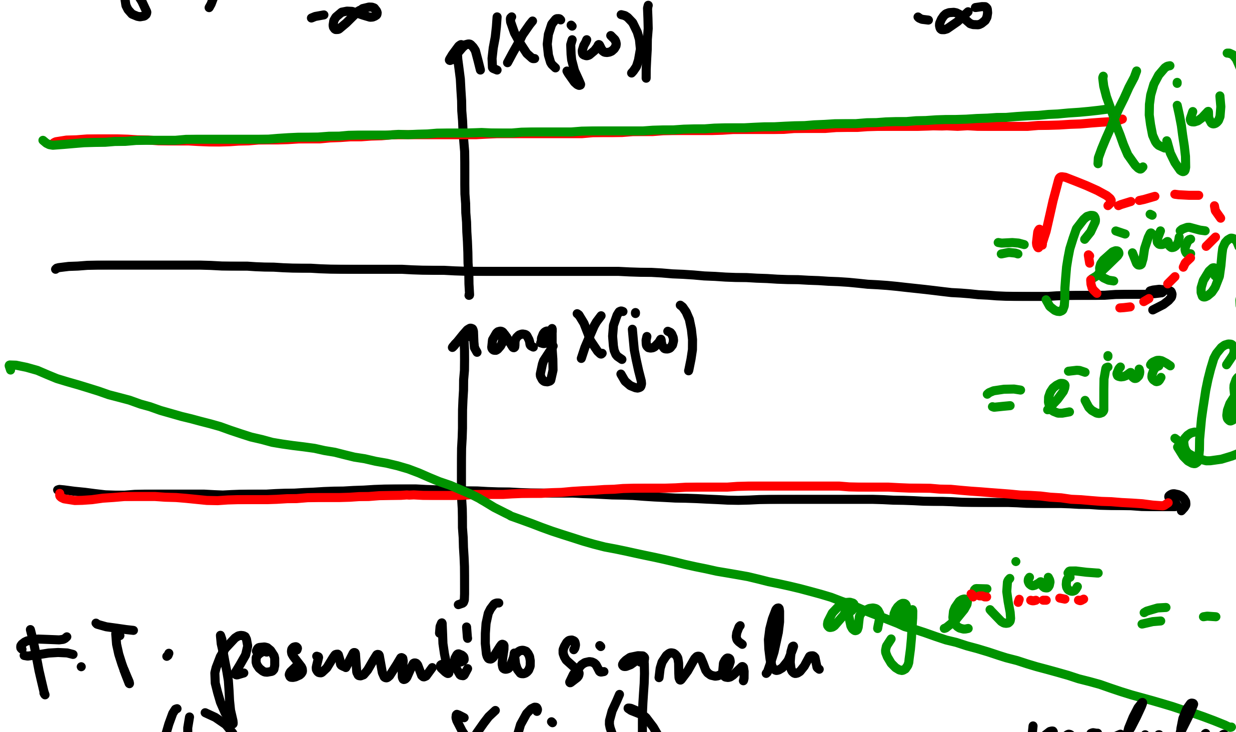


$$\int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,dt = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,dt$$

$$= \underline{x(\tau)}$$

F.T. $e^{-j\omega\tau}$

$$X(j\omega)=\int_{-\infty}^{\infty}\delta(t)e^{-j\omega t}\,dt=\int_{-\infty}^{\infty}\delta(t)\,dt=1$$



$$X(j\omega)=\int_{-\infty}^{\infty}\delta(t-\tau)e^{-j\omega t}\,dt=$$

$$=\int e^{-j\omega\tau}\delta(t-\tau)\,dt=$$

$$=e^{-j\omega\tau}\underbrace{\int\delta(t-\tau)\,dt}_{1}=e^{-j\omega\tau}$$

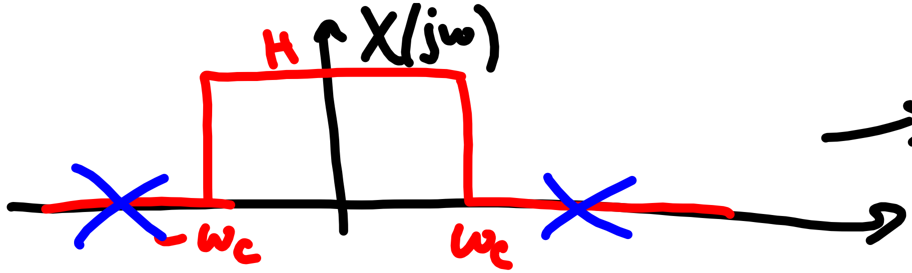
$$\operatorname{ang} e^{-j\omega\tau}=-\omega\tau$$

F.T. posunutého signálu

$x(t) \longrightarrow X(j\omega)$

$x(t-\tau) \longrightarrow X(j\omega)e^{-j\omega\tau}$

- moduly zůstanou
- argumenty
  - z kopce pro $\tau>0$
  - do kopce pro $\tau<0$



→ signál ??

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+j\omega t}\, d\omega =$$

$$\int_{-b}^{b} e^{+jxy}\, dy = 2b\, \mathrm{sinc}\, bx$$

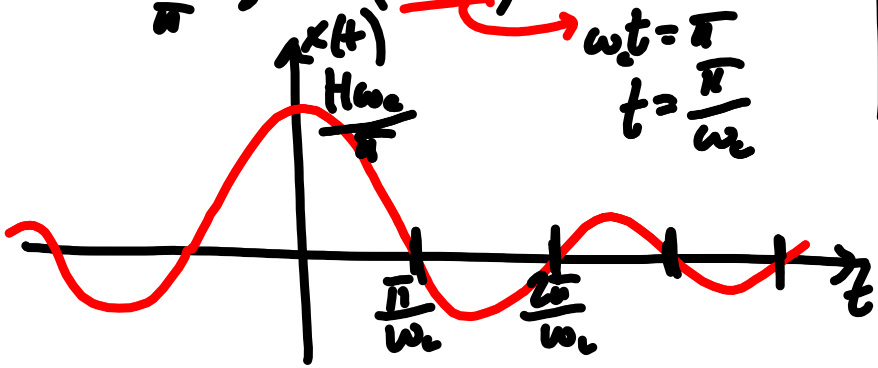
$$= \frac{H}{2\pi}\int_{-\omega_c}^{\omega_c} e^{+j\omega t}\, d\omega =$$

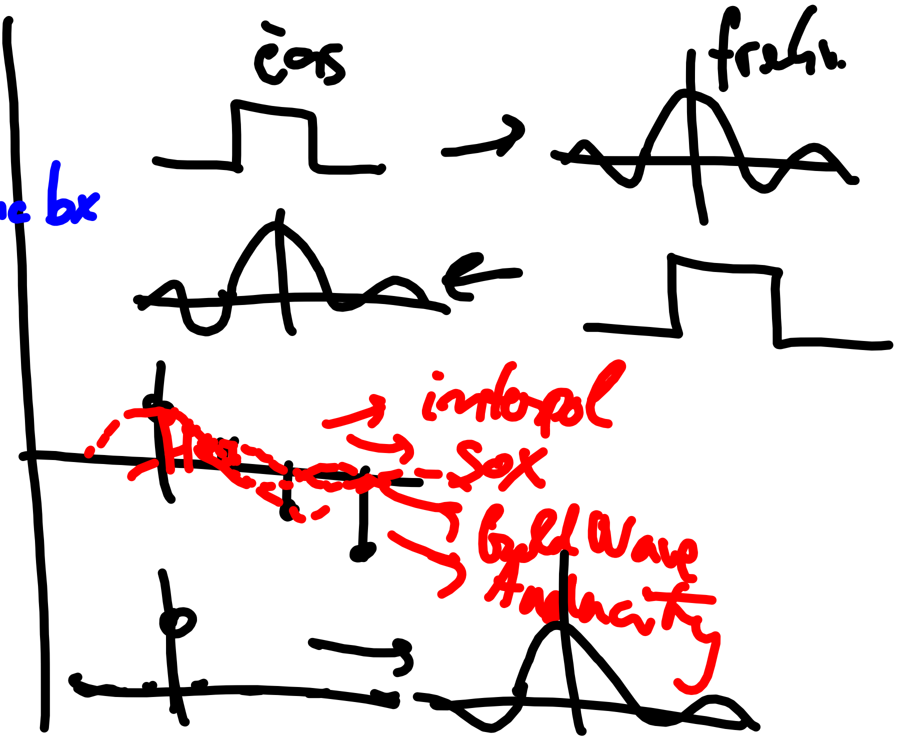
$$= \frac{H}{\cancel{2}\pi}\cancel{2}\omega_c\, \mathrm{sinc}\,(\omega_c t) =$$

$$= \frac{H\omega_c}{\pi}\, \mathrm{sinc}_c(\omega_c t)$$

$\omega_c t = \pi$

$t = \frac{\pi}{\omega_c}$





SHRNUTÍ F.T.

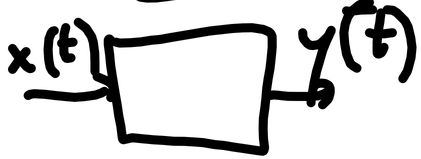
- vstup: neperiodický signál
- výstup: spektrální funkce
  def. na všech frekvencích

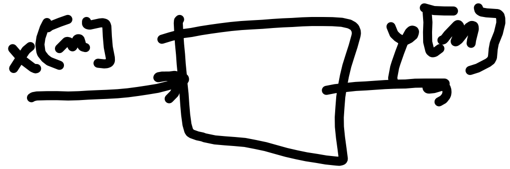
pro $x(t) \in \mathbb{R}$: $X(j\omega) = X^*(-j\omega)$

$x(t) \rightarrow X(j\omega)$

$x(t-\tau) \longrightarrow X(j\omega)e^{-j\omega\tau}$

SYSTÉMY ≈ filtry





Paměť

$$y(t) = f(x(t))$$

$a\,x(t)$

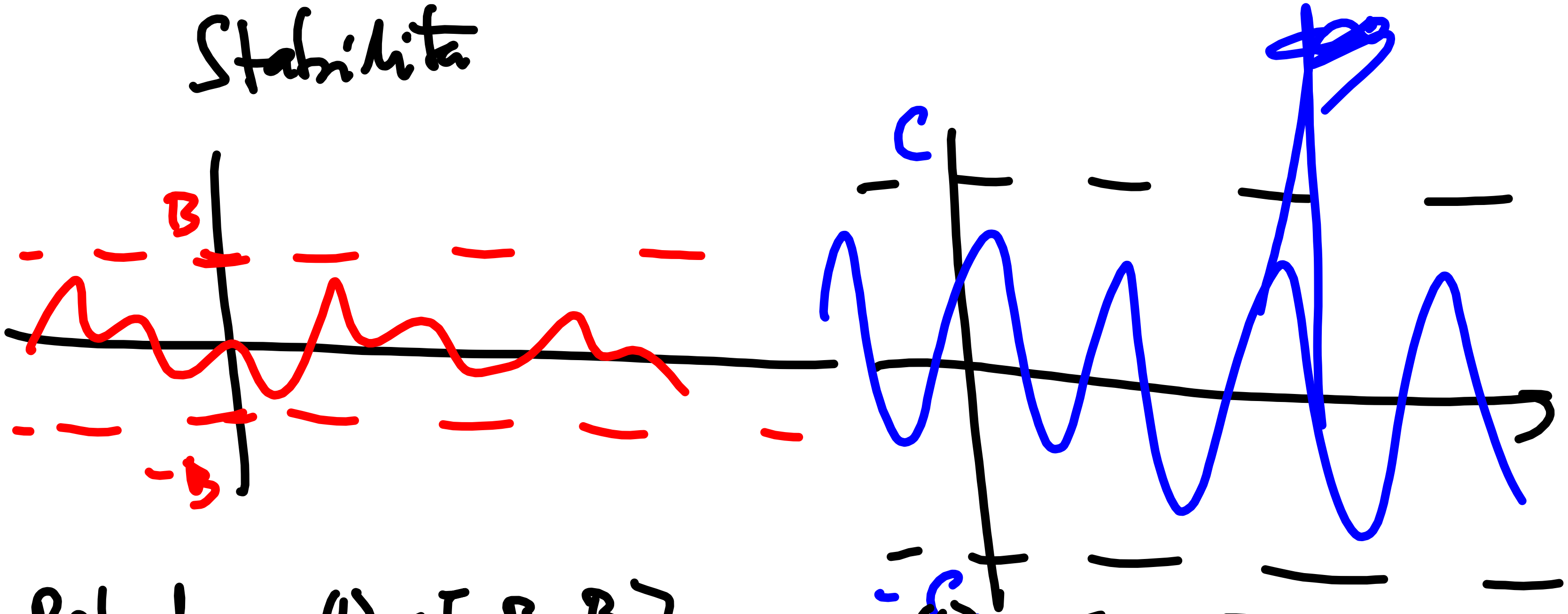
$$y[n] = 4x[n]$$

$$y[n] = \frac{x[n] + x[n-1]}{2}$$

Kauzalita

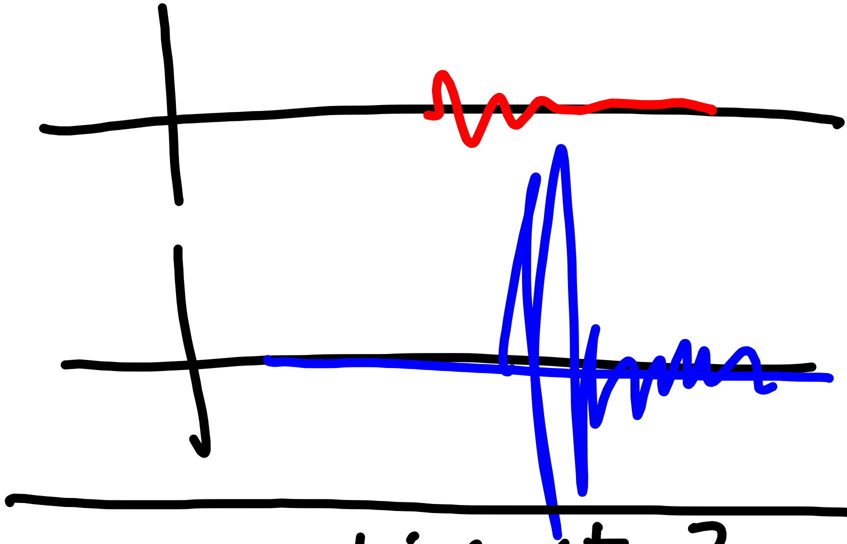
$$y(t) = f(x(u_x \leq t))$$

Stabilita



Pokud $x(t) \in [-B, B] \rightarrow y(t) \in [-C, C] \Rightarrow$ stabilní

Čas. invariantnost

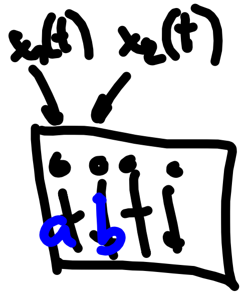


$$\left.\begin{array}{l} x(t) \to y(t) \\ x(t-\tau) \to y(t-\tau) \end{array}\right\} \text{je čas. invar.}$$

Linearita ?

$x_1(t) \to y_1(t)$

$x_2(t) \to y_2(t)$



$$a x_1(t) + b x_2(t) \to \underbrace{a y_1(t) + b y_2(t)}_{\text{lineární}}$$