Přednáška:

stř. 3.11. 9^{45}

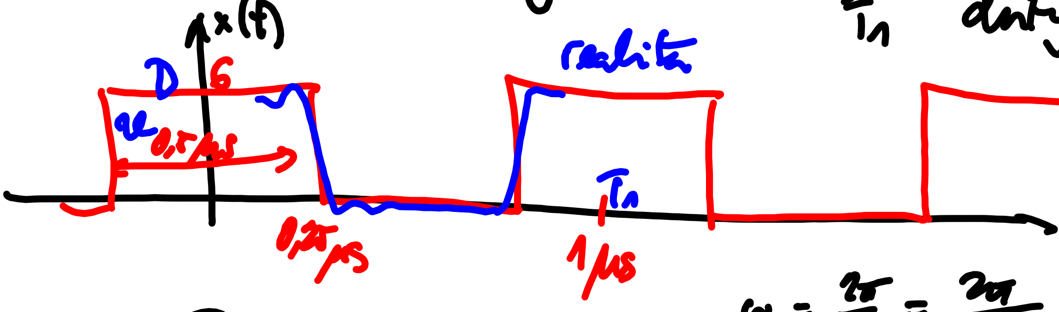
pá. 5.11. 15^{45}

COVID danger!

FŘ - posun signálu.

τ střída

$\frac{\tau}{T_1}$ duty cycle



$$\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{1\cdot10^{-6}} = 2M\pi$$

$$c_k = \frac{D\tau}{T_1}\,\mathrm{sinc}\left(\frac{\tau}{2}k\omega_1\right) \rightarrow \text{pom. funkce pro obecnou } \omega:\ \mathrm{sinc}\left(\frac{\tau}{2}\omega\right)$$

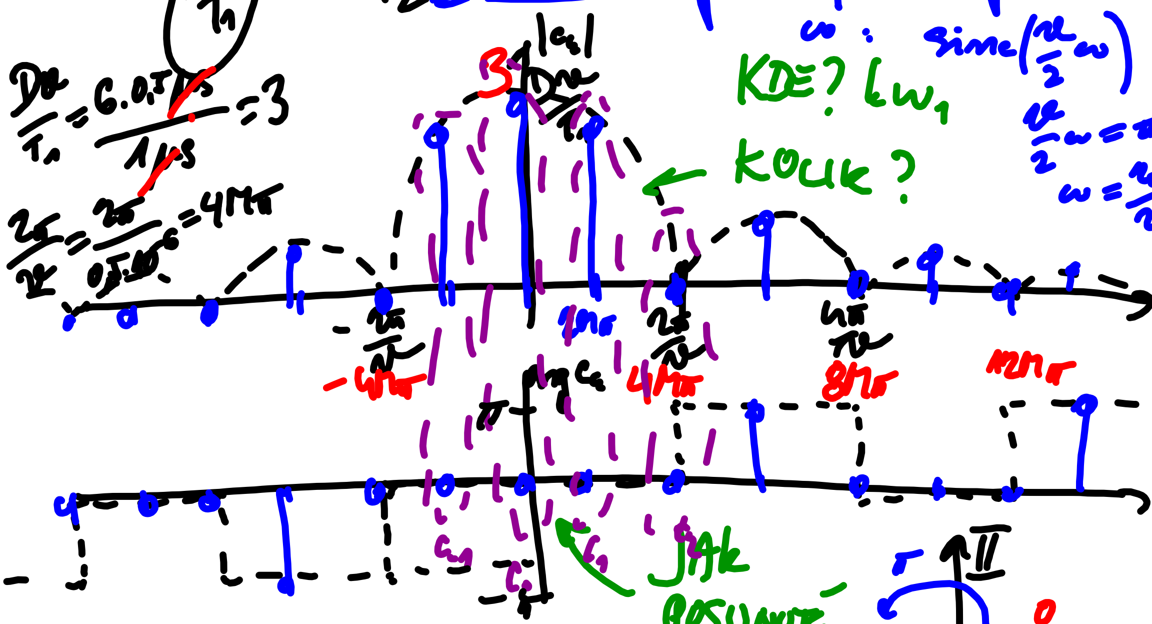
$$\frac{D\tau}{T_1} = \frac{6\cdot0{,}5\mu s}{1\mu s} = 3$$

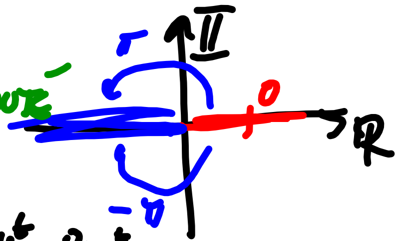
$$\frac{2\pi}{\tau} = \frac{2\pi}{0{,}5\cdot10^{-6}} = 4M\pi$$

$$\frac{\tau}{2}\omega = \pi \qquad \omega = \frac{2\pi}{\tau}$$

KDE? $k\omega_1$

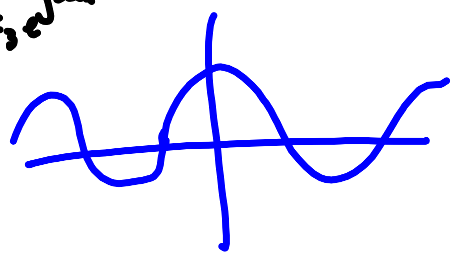
KOLIK?





$$x(t) = c_{-3}e^{-j3\omega_1 t} + \cancel{c_{-2}e^{-j2\omega_1 t}} + c_{-1}e^{-j\omega_1 t} + c_0 + c_1 e^{j\omega_1 t} + \cancel{c_2 e^{j2\omega_1 t}} + c_3 e^{j3\omega_1 t}$$

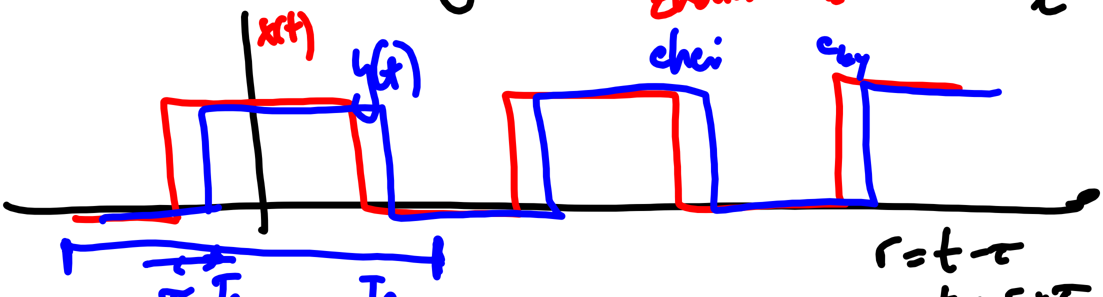
$c_{-2} = 0$ $\qquad$ $c_2 = 0$



FŘ. posunutého signálu

Známe c_k chceme c_{ky}

$e^{a+b} = e^a \cdot e^b$



$r = t - \tau$

$t = r + \tau$

$$c_{ky} = \frac{1}{T_1}\int_{T_1} y(t)\, e^{-jk\omega_1 t}\, dt = \frac{1}{T_1}\int_{T_1} x(t-\tau)\, e^{-jk\omega_1 t}\, dt =$$

$$= \frac{1}{T_1}\int_{T_1} x(r)\, e^{-jk\omega_1 (r+\tau)}\, dr = \frac{1}{T_1}\int x(r)\, e^{-jk\omega_1 r} \left(e^{-jk\omega_1 \tau}\right) dr =$$

$$= e^{-jk\omega_1\tau}\, \underbrace{\frac{1}{T_1}\int_{T_1} x(r)\, e^{-jk\omega_1 r}\, dr}_{c_k} = e^{-jk\omega_1\tau}\cdot c_k$$

$z = r\cdot e^{j\varphi}$

modul arg

$a\cdot b$

$|ab| = |a||b|$

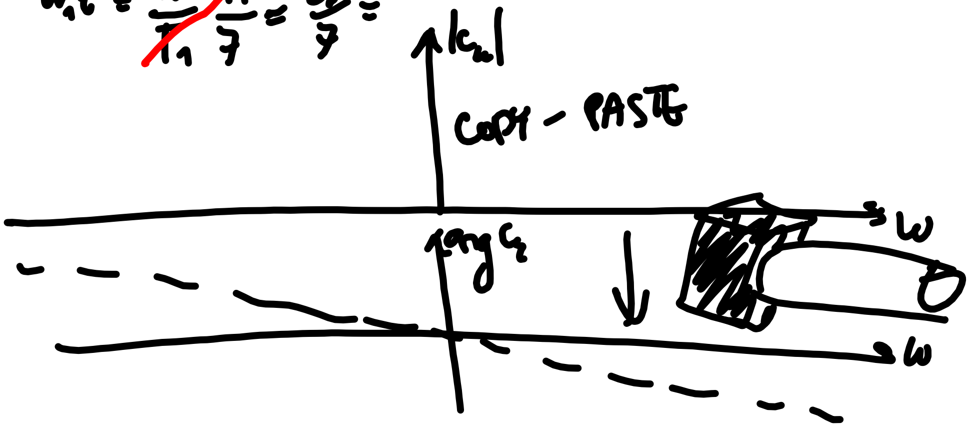
$$|c_{ky}| = \underbrace{|e^{-jk\omega_1\tau}|}_{1}\,|c_k| = |c_k|$$ moduly zůstanou stejné!

$\arg a\cdot b = \arg a + \arg b$

$$\arg c_{ky} = \arg\left(e^{-jk\omega_1\tau}\right) + \arg c_k = -k\underbrace{\omega_1\tau}_{} + \arg c_k$$

konstanta!

$$\omega_1\tau = \frac{2\pi}{\cancel{T_1}}\frac{\cancel{T_1}}{7} = \frac{2\pi}{7} =$$



SHRNUTÍ F. Ř.

vstup: periodický signál se spojitým časem.

výstup: koeficienty c_k F. řady ← "spektrum"

kde? $k\omega_1$, kde $\omega_1 = \frac{2\pi}{T_1}$

kolik? $|c_k|$

jak posunutý? $\arg c_k$

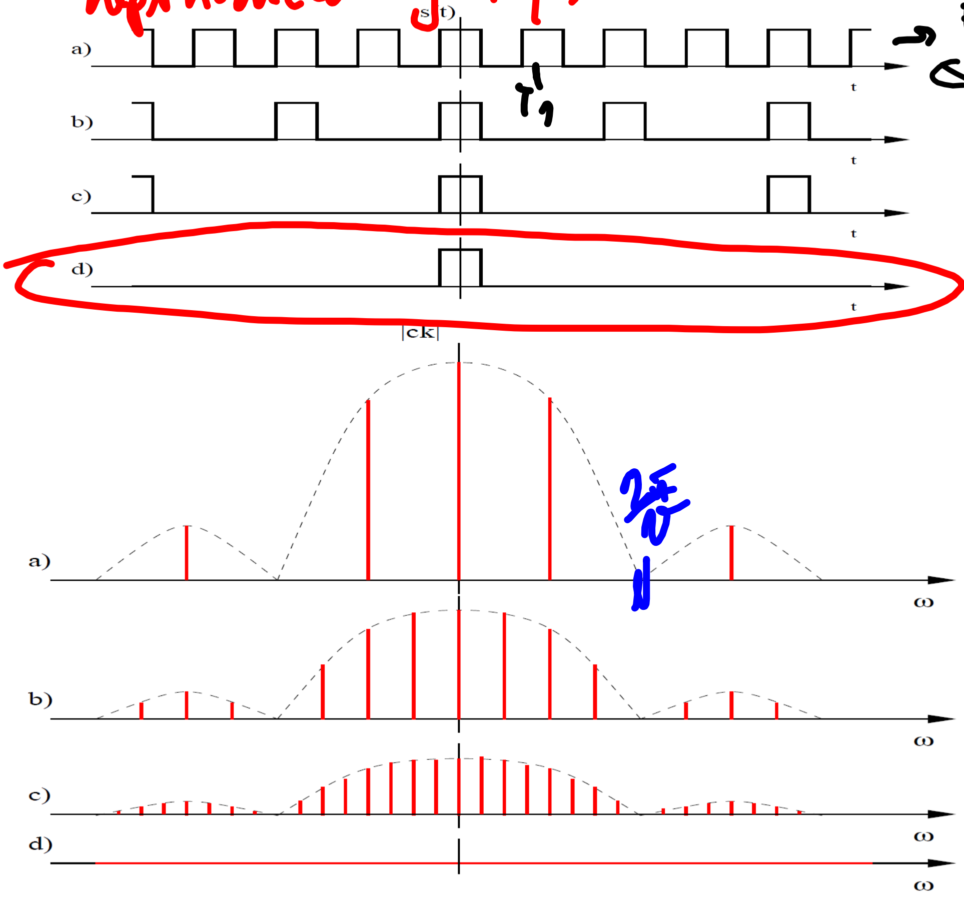
zpožděný signál: $|c_k|$ – copy-paste

$\arg c_k$ "sjíždějí z kopečka"

syntéza: vždy s páry koeficientů c_k a c_{-k}

Fourierova transformace F.T.

neperiodické signály!



→ F.Ř.

$c_k = \frac{1}{T_1}\int_{T_1} x(t) e^{-jk\omega_1 t} dt$

HOW TO HACK F.Ř. aby pracovala s neperiodickými signály?

$T_1 \to \infty$

$$c_k = \frac{1}{T_1} \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} x(t) e^{-jk\omega_1 t} \, dt$$

$T_1 = \frac{2\pi}{\omega_1}$ $\frac{1}{T_1} = \frac{\omega_1}{2\pi}$

$\boxed{\omega_1 = \frac{2\pi}{T_1} \to d\omega}$ $\frac{1}{T_1} = \frac{d\omega}{2\pi}$

$$dc = \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$$

ω - libovolná kruhová frekvence
$k\omega_1$ neexistuje!

$\boxed{c_k \to dc}$

$$2\pi \frac{dc}{d\omega} = \int_{-\infty}^{\infty} x(t) e^{-j\omega_1 t} \, dt$$

Spektrální funkce

$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt}$$

F.T.

$j\omega$?

souvislost s Laplaceovou transf.

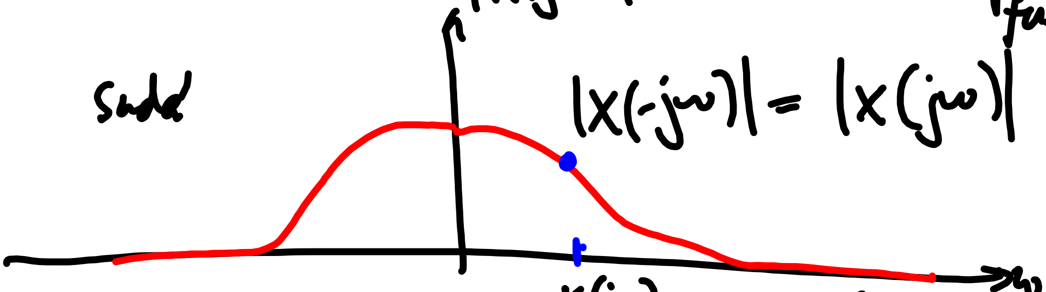
TBD later.

Jak kreslit F.T. ?

modul spek. funkce

sudá

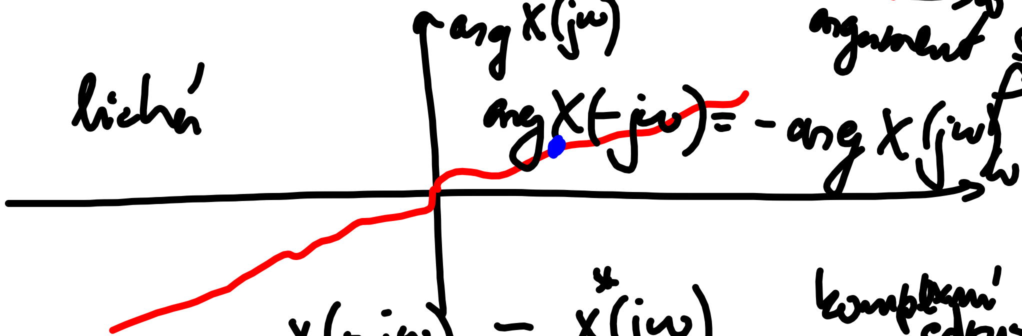
$$|X(-j\omega)| = |X(j\omega)|$$



argument spek. funkce

lichá

$$\arg X(-j\omega) = -\arg X(j\omega)$$



$$X(-j\omega) = X^*(j\omega)$$ komplexní sdružení.

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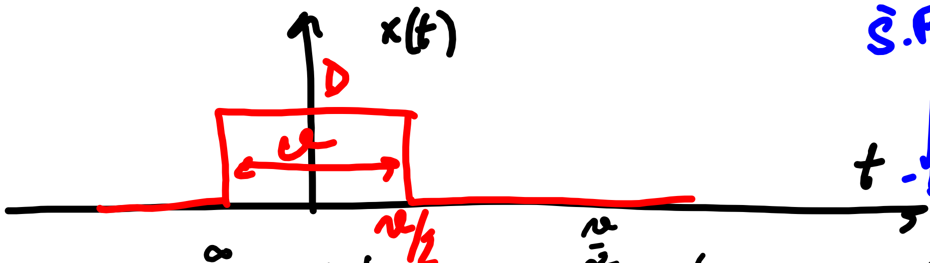
výstup = možná, nějaká konstanta · sum. operátor · vstup · $e^{\mp j \text{čas frekvence}}$

F.T.: $$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-jt\omega}\, dt \quad \longleftrightarrow \quad \text{F.Ř.: } c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jtk\omega_1}\, dt$$

IFT: $$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+jt\omega}\, d\omega} \quad \longleftrightarrow \quad \text{IFŘ: } x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{+jtk\omega_1}$$



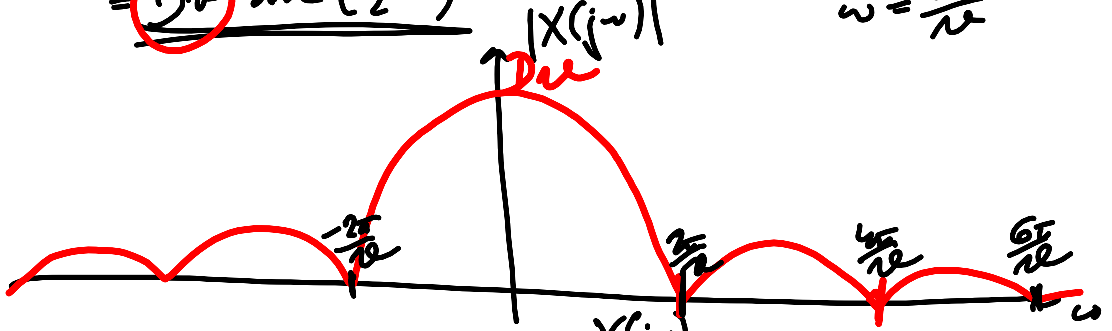
Š.P$_b$: $\int_{-b}^{b} e^{-jxy}\,dy = 2b\,\mathrm{sinc}\,bx$

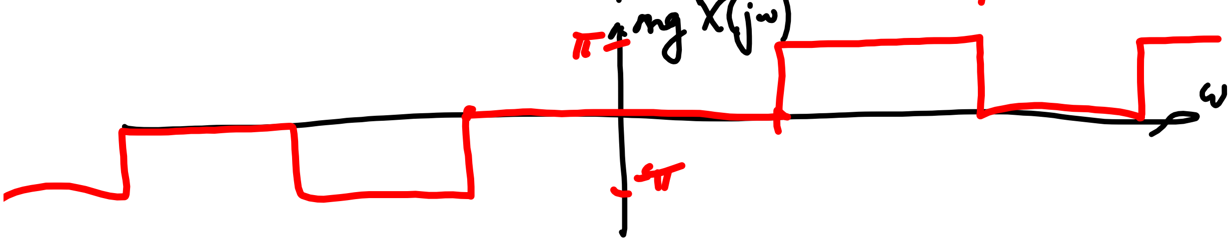
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt = D\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}\,dt = D\cdot 2\frac{\tau}{2}\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right) =$$

$$= 1\,\tau\;\mathrm{sinc}\left(\frac{\tau}{2}\omega\right)$$

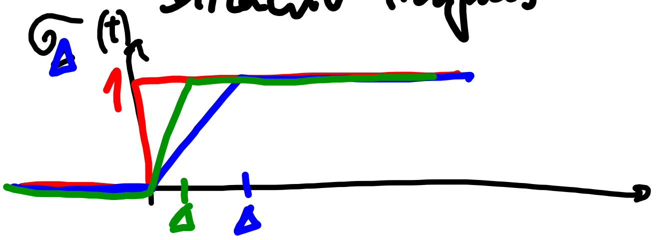
$\frac{\tau}{2}\omega = \pi$

$\omega = \frac{2\pi}{\tau}$



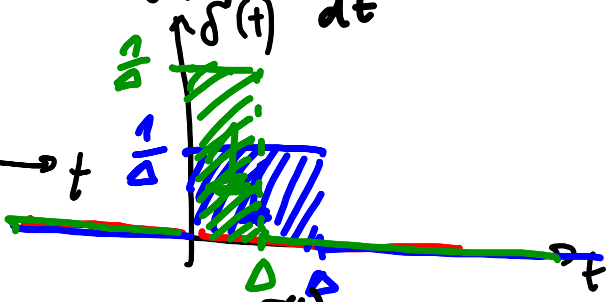


Diracův impuls

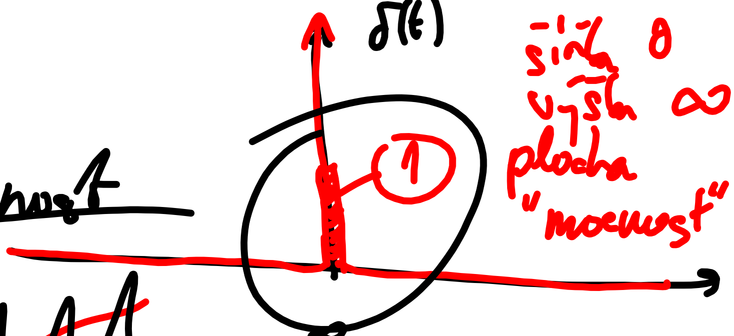


jednotkový skok

$$\delta(t) = \frac{d\sigma(t)}{dt}$$

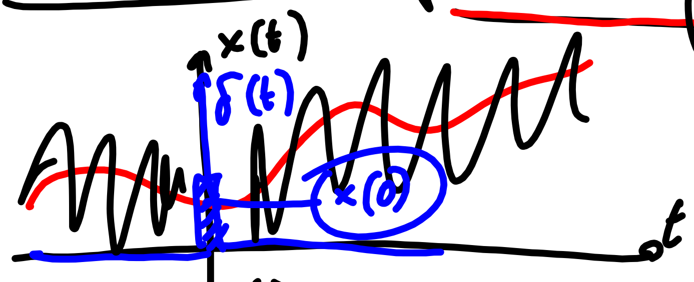


$\frac{1}{\Delta}t$

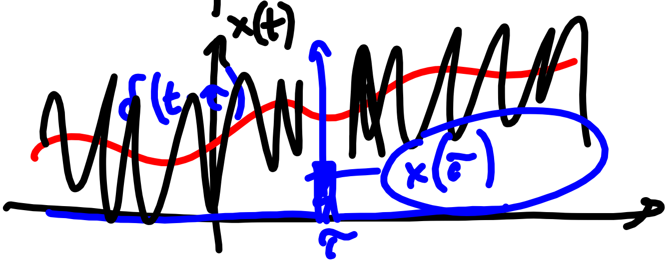


šířka 0
výška ∞
plocha "mocnost" 1

vzorkovací schopnost

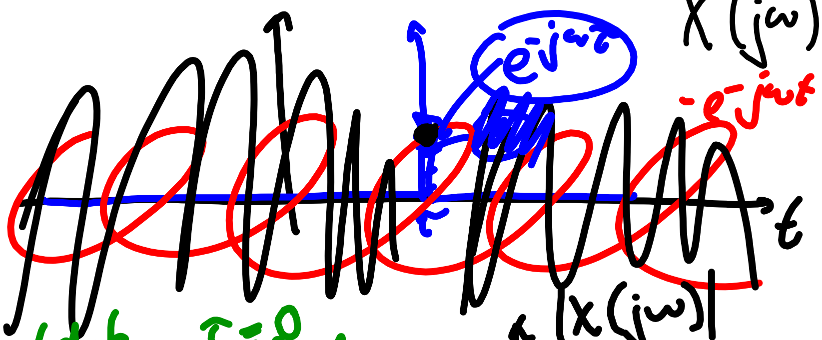


$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$$



$$\int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,dt = x(\tau)$$

F.T. $\delta(t-\tau)$?



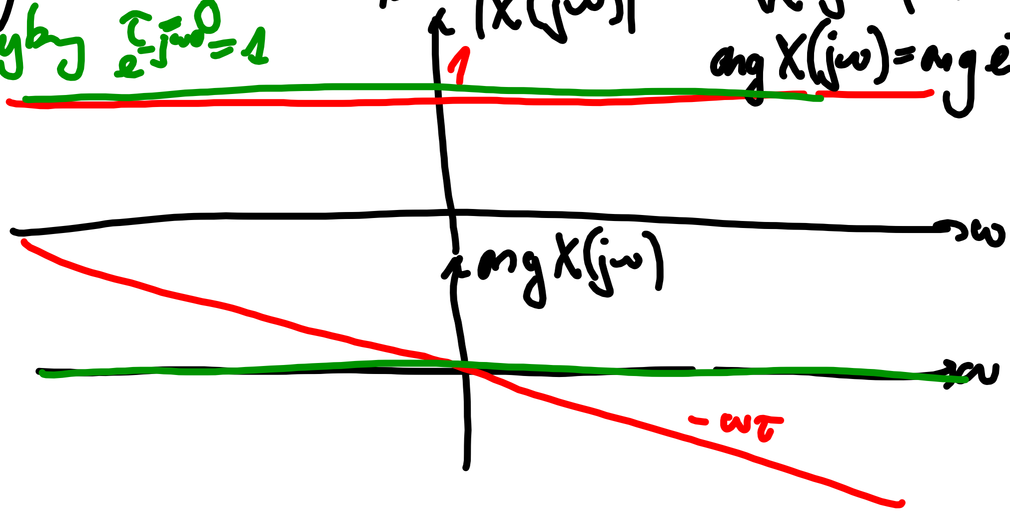
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt =$$

$$= e^{-j\omega\tau}$$

$$|X(j\omega)| = |e^{-j\omega\tau}| = 1$$

$$\text{ang}\, X(j\omega) = \text{ang}\, e^{-j\omega\tau} = -\omega\tau$$

kdyby $\tau = 0$ $e^{-j\omega 0} = 1$



Signál odpovídající obdélníkovému spektru

IFT: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+j\omega t}\, d\omega =$

$= \frac{1}{2\pi} H \int_{-\omega_c}^{\omega_c} e^{j\omega t}\, d\omega =$

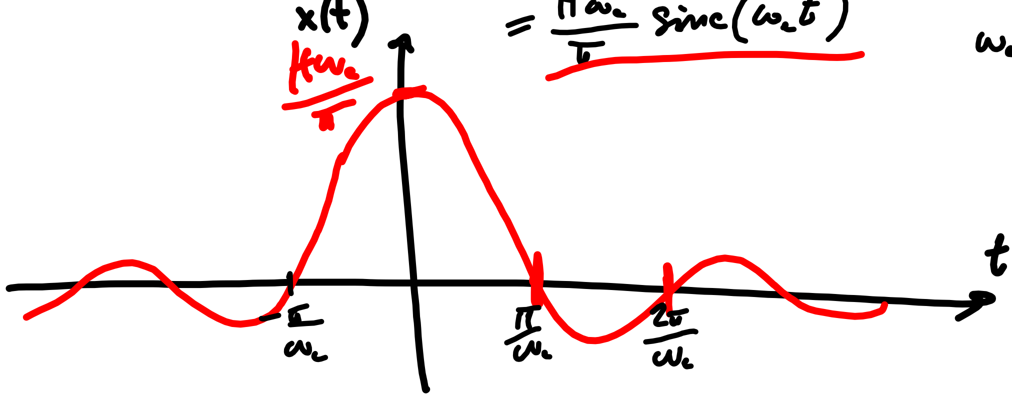
z.p. $\int_{-b}^{b} e^{\pm jxy}\, dy = 2b \,\mathrm{sinc}\, bx$

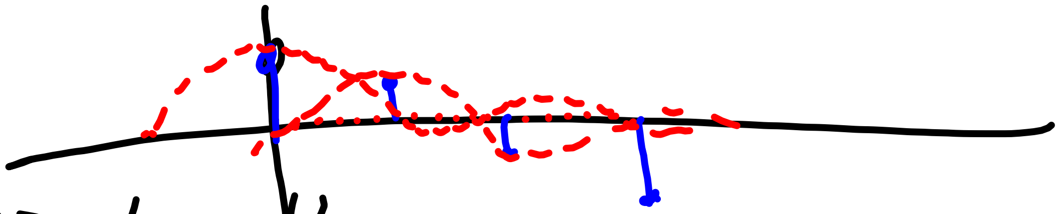
$= \frac{H}{2\pi} 2\omega_c \,\mathrm{sinc}(\omega_c t) =$

$= \frac{H\omega_c}{\pi} \mathrm{sinc}(\omega_c t)$

$\omega_c t = \pi$

$t = \frac{\pi}{\omega_c}$





F.T. shrnutí:

vstup: neperiodický signál se spoj. časem.

výstup: spektrální funkce $X(j\omega)$

kde? všude

kolik? $|X(j\omega)|$

jak posunuté? $\arg X(j\omega)$

$X(-j\omega) = X^*(j\omega)$

$x(t) \longrightarrow X(j\omega)$

$x(t-\tau) \longrightarrow X(j\omega)\cdot e^{-j\omega\tau}$

↗ moduly se nezmění (copy-paste)

↘ argumenty $-\omega\tau$

Systémy

časová invariantnost $x(t) \rightarrow y(t)$

$x(t-\tau) \rightarrow y(t-\tau)$

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linearita $x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)$

---

kauzalita $\left.\begin{array}{l} x(t) \\ x(\tau < t) \\ y(\tau < t) \end{array}\right\} \rightarrow y(t)$

---

stabilita "bounded input → bounded output"

$x(t) \in \langle -B, B \rangle$ $y(t) \in \langle -C, C \rangle$

známo

musím najít → stabilní

∞ → nestabilní.