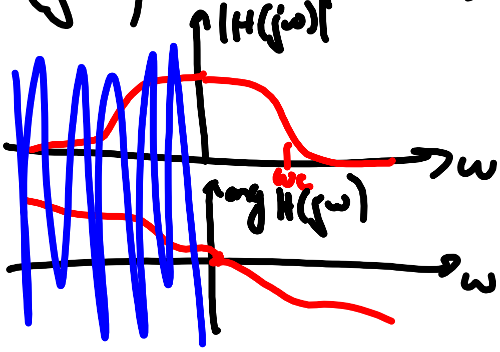
Systémy se spoj. časem

$h(t)$ impulsní odezva

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$$

$H(j\omega)$ (komplexní) kmitočtová charakteristika



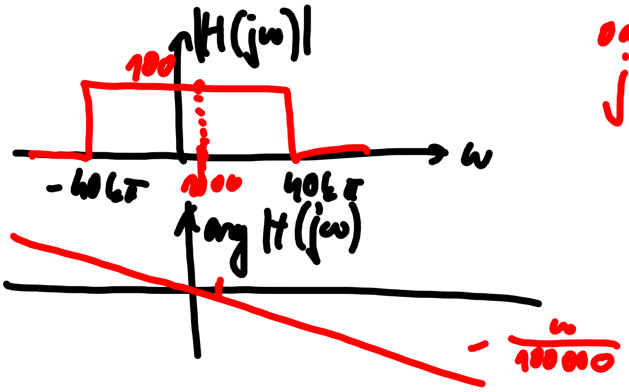
$$H(j\omega) = F.T.\left(h(t)\right)$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$H(-j\omega) = H^*(j\omega)$$

modul sudý

arg. opačné



od 0 do 20 GHz zesiluje 100x
jinde 0
Ideální HiFi zesilovač

zesílení 100
fáz. posun $-\frac{1000}{100\,000} = -\frac{1}{100}$

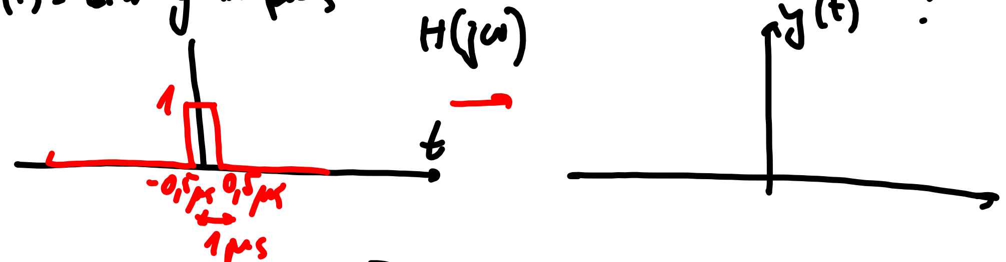
vstup: $x(t) = 10\cos\left(1000t + \frac{\pi}{2}\right)$

$y(t) = 1000 \cos\left(1000t + \frac{\pi}{2} - \frac{1}{100}\right)$ zesílená a zpožděná cos.

$x(t) = 10\cos(100000t)$ $y(t) = 0$

$x(t) = 10e^{j1000t} = 1000e^{j(1000t - \frac{1}{100})} = \underbrace{1000e^{-j\frac{1}{100}}}_{\text{konst.}} e^{j1000t}$

$x(t)$ = krátký impuls



option 1: $H(j\omega) \xrightarrow{IFT} h(t)$ $\quad y(t) = x(t) * h(t)$ hm, ke.

option 2: 1. $x(t) \xrightarrow{F.T.} X(j\omega)$

2. $Y(j\omega) = X(j\omega)\, H(j\omega)$ násobení!

3. $Y(j\omega) \xrightarrow{IFT} y(t)$

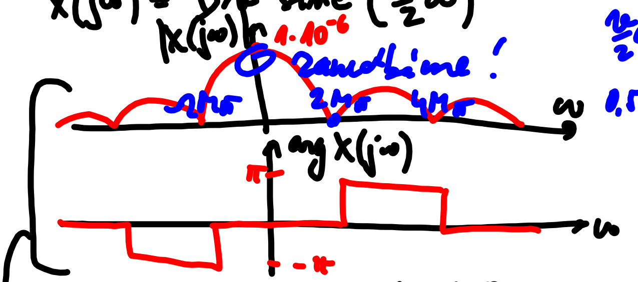
krok 1

$X(j\omega) = D\tau\ \mathrm{sinc}\left(\frac{\tau}{2}\omega\right)$ viz minule

$\frac{\tau}{2}\omega = \pi$

$0{,}5\cdot10^{-6}\omega = \pi$

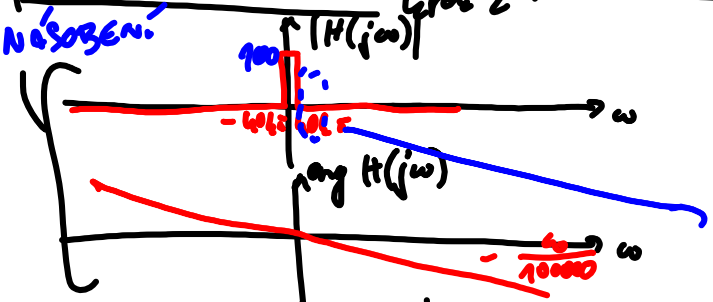
$\omega = 2\pi\cdot10^{+6}$



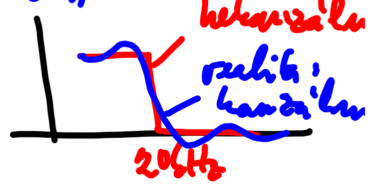
krok 2

NÁSOBENÍ

pozor na měřítko, zesilovač propustí od $-40k\pi$ do $40k\pi$



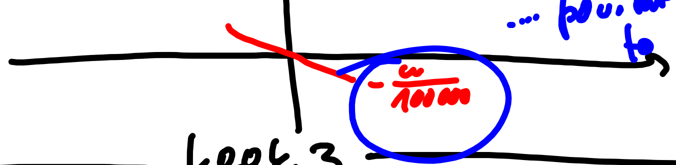
zoom



... použít jí to ↑

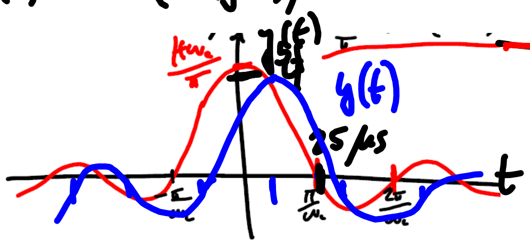
$u_1(t) \to U_1(j\omega)$

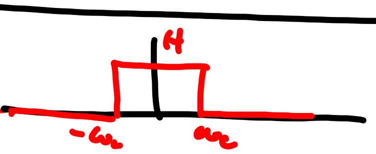
$u_1(t-\tau) \to U_1(j\omega)\, e^{-j\omega\tau}$



krok 3

$y(t) = IFT(Y(j\omega))$





$\frac{H\omega_c}{\pi} = \frac{10^{-4}\cdot 40\cdot10^{3}\pi}{\pi} = 4$

$\frac{\pi}{\omega_c} = \frac{\cancel{\pi}}{40\cdot10^{3}\cancel{\pi}} = 0{,}025\cdot10^{-3} = 25\,\mu s$

$y(t)$ zpožděný oproti $y_{bf}(t)$ "bez fáze" o $10\,\mu s$.

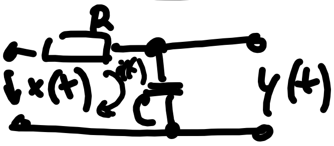
$h(t) \xrightarrow{F.T.} H(j\omega)$

Diferenciální rovnice

$$b_0 x(t) + b_1 \frac{dx(t)}{dt} + \dots + b_q \frac{d^q x(t)}{dt^q} = a_0 y(t) + a_1 \frac{dy(t)}{dt} + \dots a_p \frac{d^p y(t)}{dt^p}$$

$$\sum_{k=0}^{q} b_k \frac{d^k x(t)}{dt^k} = \sum_{k=0}^{p} a_k \frac{d^k y(t)}{dt^k} \qquad \mathcal{L}$$

Diferenciální rovnice



$$i(t) = \frac{x(t) - y(t)}{R}$$

$$i(t) = C \frac{dy(t)}{dt}$$

$$\frac{x(t) - y(t)}{R} = C \frac{dy(t)}{dt}$$

$$x(t) = y(t) + RC \frac{dy(t)}{dt}$$

$q = 0$, $b_0 = 1$; $p = 1$, $a_0 = 1$, $a_1 = RC$

np.freqs(B, A, ω)

Laplaceova transformace

$$\boxed{H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt} \qquad \boxed{H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt}$$

F.T. slovníček L.T.: s je v celé komplexní rovině

$x(t) \rightarrow X(s)$

$a\,x(t) \rightarrow a\,X(s)$

$\frac{d^k x(t)}{dt^k} \rightarrow X(s) s^k$

toto je důvod pro zápis $H(j\omega)$!

$$b_0 X(s) + b_1 X(s) s + \dots b_q X(s) s^q = a_0 Y(s) + a_1 Y(s) s + \dots + a_p Y(s) s^p$$

Přenosová funkce: $H(s) = \frac{Y(s)}{X(s)}$

$$X(s)(b_0 + b_1 s + \dots + b_q s^q) = Y(s)(a_0 + a_1 s + \dots + a_p s^p)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_0 + b_1 s + \dots + b_q s^q}{a_0 + a_1 s + \dots + a_p s^p} = \frac{\sum_{k=0}^{q} b_k s^k}{\sum_{k=0}^{p} a_k s^k}$$

podíl (lomená funkce) dvou polynomů

$$X(s) = Y(s) + RC\,Y(s) s$$

$$X(s) = Y(s)(1 + RCs)$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{1}{1 + RCs}$$

$$H(j\omega) = \frac{1}{1 + RCj\omega}$$

Přechod k frekv. char.: $s \longrightarrow j\omega$

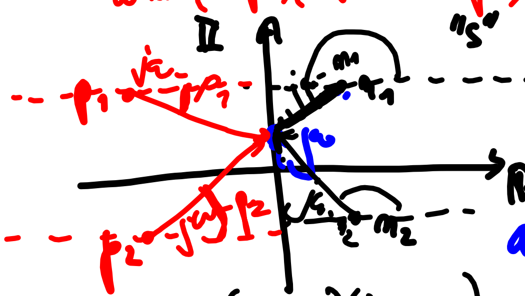
$$H(j\omega) = H(s)\Big|_{s=j\omega} = \frac{\sum b_k (j\omega)^k}{\sum a_k (j\omega)^k}$$

Faktorizace – rozklad polynomů na kořeny.

$$H(s) = \frac{const\,(s - n_1)(s - n_2)\dots(s - n_q)}{const\,(s - p_1)(s - p_2)\dots(s - p_p)}$$

n - kořeny čitatele - nulové body

p - kořeny jmenovatele - póly

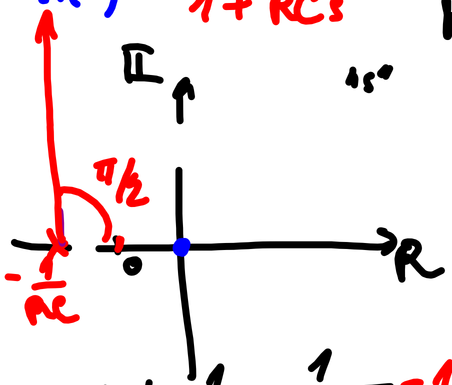


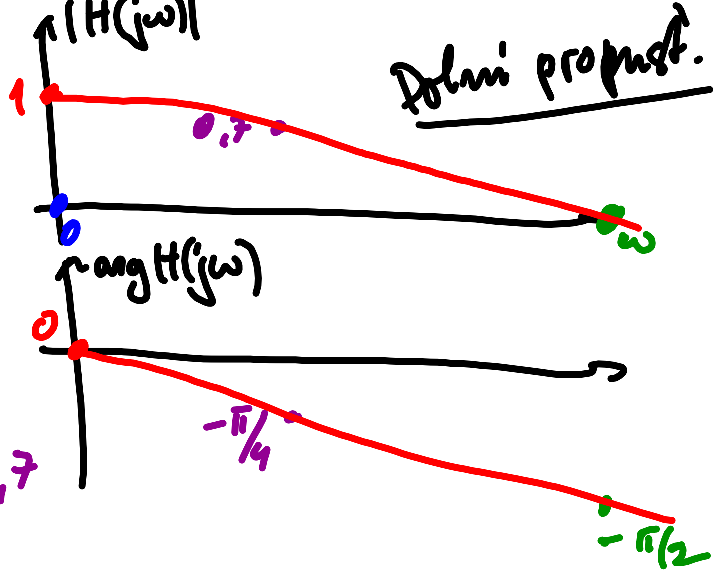
$$|H(j\omega)| = const \frac{\text{součin délek černých vektorů}}{\text{součin délek červených vektorů}}$$

$\arg H(j\omega)$ = součet úhlů černých vektorů − součet úhlů červených vektorů

$$H(j\omega) = \frac{const\,(j\omega - n_1)(j\omega - n_2)}{const\,(j\omega - p_1)(j\omega - p_2)}$$

$$H(s) = \frac{1}{1+RCs} = \frac{1}{RC\left(\frac{1}{RC}+s\right)} = \frac{1}{RC\left(s-\left(-\frac{1}{RC}\right)\right)}$$





Dolní propust.

$\omega = 0$ $\quad |H| = \frac{\frac{1}{RC}}{\frac{1}{RC}} = 1$ $\quad \arg H = 0$

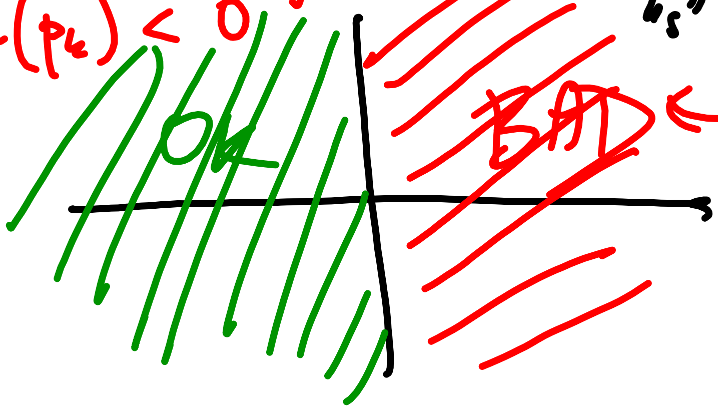
$\omega = \infty$ $\quad |H| = \frac{\frac{1}{RC}}{\infty} = 0$ $\quad \arg H = -\pi/2$

$\omega = \frac{1}{RC}$ $\quad |H| = \frac{1}{RC} \cdot \frac{1}{\sqrt{2} \cdot \frac{1}{RC}} = 0,7$ $\quad \arg H = -\frac{\pi}{4}$

Stabilita systému

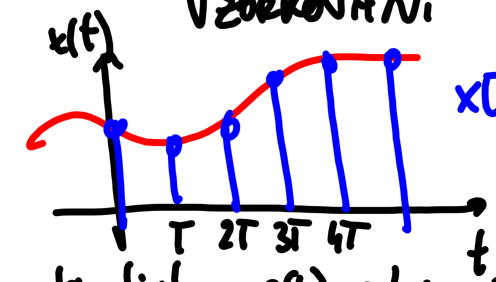
koeficienty výstupní části $a_k \longrightarrow$ póly p_k

$\mathcal{R}(p_k) < 0$



toto pro systémy se spojitým časem!

VZORKOVÁNÍ



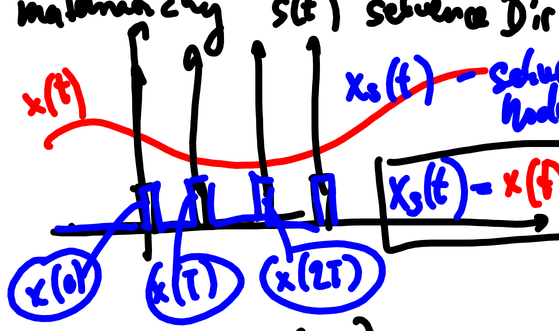
t - spojitý čas [s]

n - diskrétní čas [ ]

vzorkovací frekvence F_s

— „ — perioda $T = \frac{1}{F_s}$

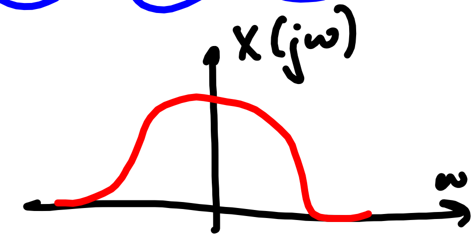
matematicky s(t) sekvence Dir. impulsů



$x_s(t)$ - sekvence Dir. impulsů s mocností dle hodnot původního signálu.

$\boxed{x_s(t) = x(t)\cdot s(t)}$ v čase

Spektrum navzorkovaného signálu?



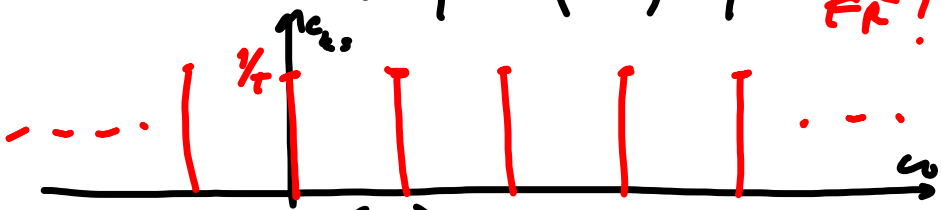
spektrum vzorkovacího signálu:

$\omega_0 = \frac{2\pi}{T}$

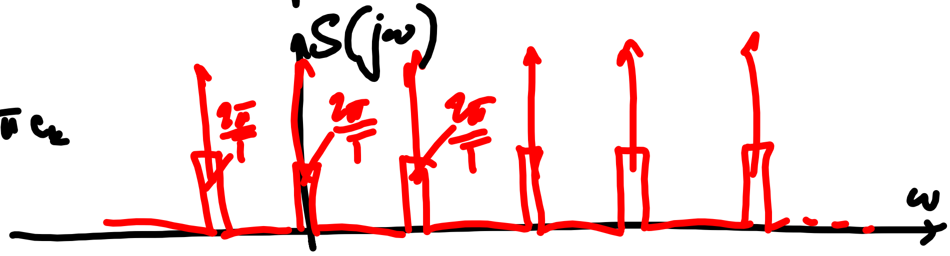
$X_s(j\omega) = X(j\omega) * S(j\omega)$

$c_k = \frac{D\vartheta}{T}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right) = \frac{\cancel{\frac{1}{\vartheta}\vartheta}}{T}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\frac{2\pi}{T}\right)$

$\vartheta \to 0$ $\quad c_k = \frac{1}{T}\,\mathrm{sinc}(0\ldots) = \frac{1}{T}$ nekonečná ŘF!



FŘ → FT
Dir. impulsy s hodnotami $2\pi c_k$ tam, kde ležely koeficienty FŘ



konvoluce ve spektru $X_s(\omega) = X(\omega) * S(\omega) =$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\tilde{\omega})\, S(\omega - \tilde{\omega})\, d\tilde{\omega}$$