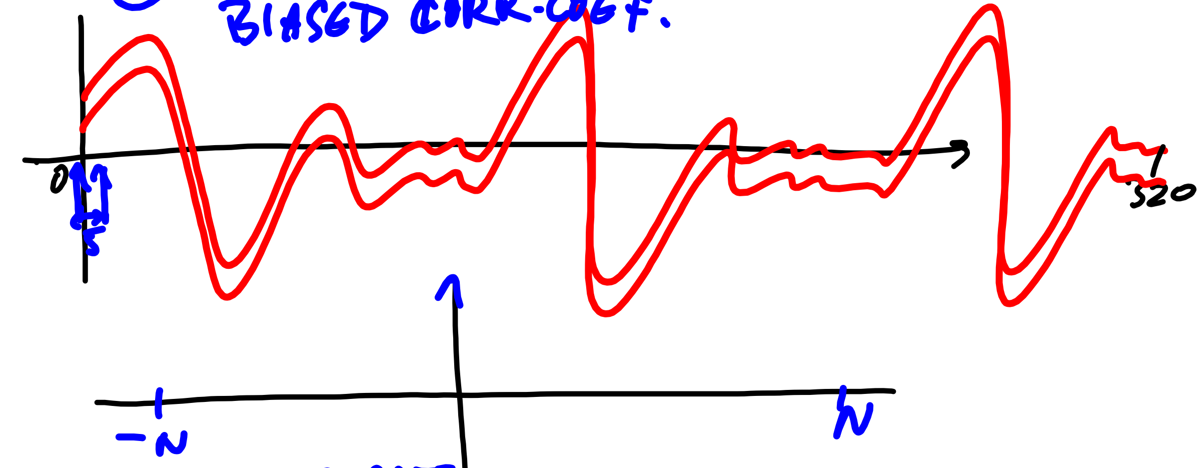
$$R[k] = \frac{1}{N} \sum x[n]\, x[n+k]$$

5 3 -1 2 5 -3 7
5 3 -1 2 5 -3 7

BIASED CORR. COEF.



→ UNBIASED CORR. COEFF.

$$R[k] = \frac{1}{N-|k|} \sum x[n]\, x[n-k]$$

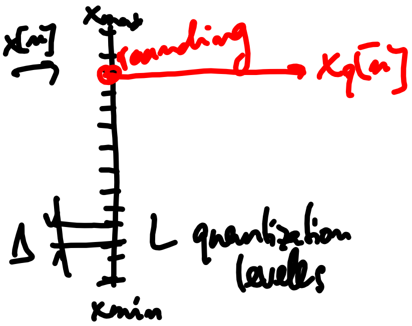
$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n] = R[0] = D$$ for signals with $a=0$

$$= D + a^2$$ if $a \neq 0$

Quantization

quantization step $\Delta = \frac{x_{max}-x_{min}}{L}$

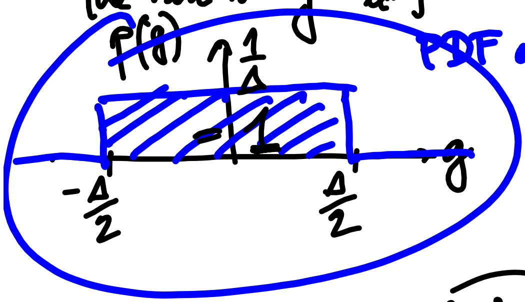
b bits $L = 2^b$



How does the quantization harm the signal ??

$$\boxed{SNR} = 10 \cdot \log_{10} \frac{\text{power of useful signal}}{\text{power of noise}} \quad [dB]$$

the bad thing: $e[n] = x[n] - x_q[n]$ quantization error, random signal



$$P_e = D = \int_{-\infty}^{\infty} p(g)(g-a)^2\,dg = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} g^2\,dg = \frac{1}{\Delta}\left[\frac{1}{3}g^3\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{\Delta}\left[\frac{\Delta^3}{3\cdot 8} - \frac{(-\Delta)^3}{3\cdot 8}\right] =$$

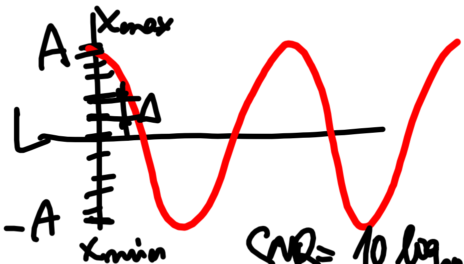
$$= \frac{1}{\Delta}\,\frac{2\cdot\Delta^3}{3\cdot 8} = \frac{\Delta^2}{12} \leftarrow \text{BAD}$$

Cosine at the input, full range of the quantizer - useful signal

$2A = L\Delta$ $\quad A = \frac{L\Delta}{2}$ $\quad C_{ef} = \frac{A}{\sqrt{2}}$

$$P = \frac{A^2}{2} = \frac{\frac{L^2\Delta^2}{2^2}}{2} = \frac{L^2\Delta^2}{8} \leftarrow \text{good.}$$

$L = 2^b \leftarrow$ bits



$$SNR = 10\log_{10}\frac{\frac{L^2\cancel{\Delta^2}}{8}}{\frac{\cancel{\Delta^2}}{12}} = 10\log_{10}\frac{3}{2}L^2 =$$

$\log a\cdot b = \log a + \log b$

$\log a^b = b\log a$

$$= 10\log_{10}\frac{3}{2}(2^b)^2 = \underbrace{10\log_{10}\frac{3}{2}}_{1{,}8} + 10\log 2^{2b} =$$

$$= 1{,}8 + b\cdot 2\cdot 10\log 2 = 1{,}8 + 6b$$

CD player

96 dB

Frequency analysis

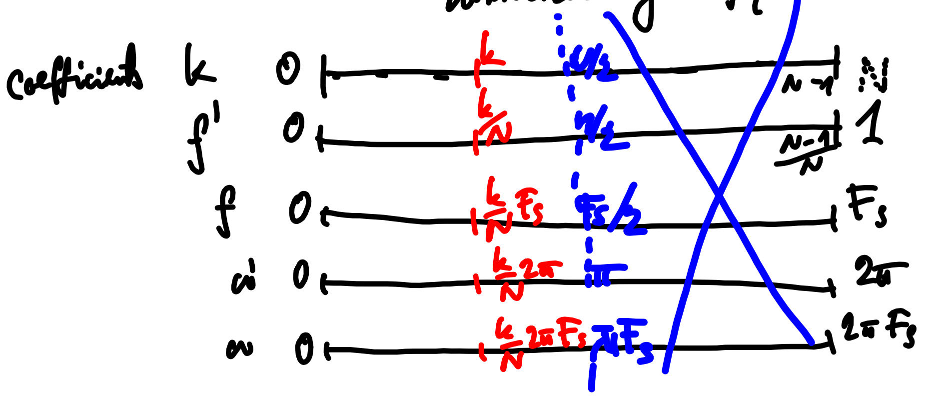
output = magnitude & const. sum. of input $e^{\pm j\,\text{time frequency}}$

− analysis: time → freq.
+ synthesis: freq → time

time	frequency
periodic	discrete (coefficients, no function!)
discrete	periodic

frequencies: analog f [Hz]; angular $\omega = 2\pi f$ [rad/s]
digital: normalized freq. f' [ ]; normalized angular freq. ω' [rad]

sampling: $f' = \frac{f}{F_s}$, $\omega' = \frac{\omega}{F_s}$



	ANALOG	DIGITAL
periodic	Fourier series (T_1 is period) $c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\,dt$, $\omega_1 = \frac{2\pi}{T_1}$ (fundamental freq.); $x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{+jk\omega_1 t}$	Discrete Fourier Series (DFS) (N is period) $\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]\, e^{-j\frac{2\pi}{N}kn}$ ($\frac{2\pi}{N}$: fundamental freq.); $\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}[k]\, e^{+j\frac{2\pi}{N}kn}$
non-periodic	Fourier transform (spectral function) $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt$; $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+j\omega t}\,d\omega$	Discrete-time Fourier transf. DTFT $\tilde{X}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$; $x[n] = \frac{1}{2\pi}\int_0^{2\pi} \tilde{X}(e^{j\omega})\, e^{+j\omega n}\,d\omega$

Discrete Fourier transform DFT without any periodicity!
take DFS, delete "~"

FILTERING → freq. responce, stability

ANALOG

scheme (circuit) → differential equation

$$\sum_{k=0}^{Q} b_k \frac{d x^k(t)}{dt^k} = \sum_{k=0}^{P} a_k \frac{d y^k(t)}{dt^k}$$

DIGITAL

scheme / code → difference equation

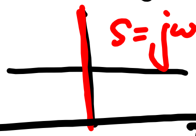
$$y[n] = \sum_{k=0}^{Q} b_k x[n-k] - \sum_{k=1}^{P} a_k y[n-k]$$

Laplace transform

F.T.

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt \qquad X(j\omega) = \int x(t) e^{-j\omega t}\, dt$$

$$\frac{d^k x(t)}{dt^k} \to X(s) s^k$$

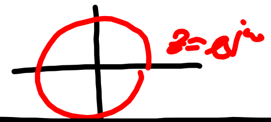


z-transform.

D.T.F.T.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad X(e^{j\omega}) = \sum x[n] e^{-j\omega n}$$

$$x[n-k] \to X(z) z^{-k}$$



TRANSFER FUNCTION : OUT / IN

$$H(s) = \frac{Y(s)}{X(s)}$$

$$\sum_{k=0}^{Q} b_k X(s) s^k = \sum_{k=0}^{P} a_k Y(s) s^k$$

$$X(s) \sum b_k s^k = Y(s) \sum a_k s^k$$

$$H(s) = \frac{\sum b_k s^k}{\sum a_k s^k}$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = \sum b_k X(z) z^{-k} - \sum a_k Y(z) z^{-k}$$

$$Y(z) \left[1 + \sum a_k z^{-k}\right] = X(z) \sum b_k z^{-k}$$

$$H(z) = \frac{\sum b_k z^{-k}}{1 + \sum a_k z^{-k}}$$

FREQ. CHAR.

$$H(j\omega) = \frac{\sum b_k (j\omega)^k}{\sum a_k (j\omega)^k}$$

np. freqs

$$H(e^{j\omega}) = \frac{\sum b_k e^{-j\omega k}}{1 + \sum a_k e^{-j\omega k}}$$

np. freqz

Factorization of polynomials

$$H(s) = \frac{b_Q (s-n_1)(s-n_2)\dots(s-n_Q)}{a_P (s-p_1)(s-p_2)\dots(s-p_P)}$$

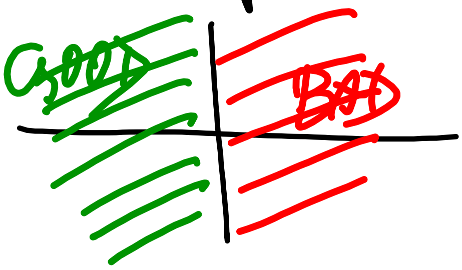
zero points

poles

$$H(z) = \frac{b_0 z^{-Q} (z-n_1)(z-n_2)\dots(z-n_Q)}{z^{P} (z-p_1)(z-p_2)\dots(z-p_P)}$$

Stability:

$$\forall\ \mathrm{Real}(p_k) < 0$$



$$\forall\ |p_k| < 1$$