

zakladni

$$\xi[0] \quad \xi[1] \quad \xi[2] \quad \dots \quad \xi[n] \quad \dots \quad \xi[N-1]$$

$$\xi_1[n] \quad \xi_2[n] \quad \xi_\omega[n] \quad \dots \quad \xi_\omega[n]$$

$$\begin{aligned} \xi[n] &\in \mathcal{R} & \xi[n] &\in [h_1, h_2, \dots, h_H] \\ \xi[n] &\in [h_1, h_2, \dots, h_H] = [0, 1, 2, \dots, 36] \end{aligned}$$

distrib fce

$$F(x, n) = \mathcal{P}\{\xi[n] < x\},$$

proba

$$\text{probability} = \frac{\text{count}}{\text{total}}$$

$$\hat{F}(x, n) = \frac{\text{count}(\xi_\omega[n] < x)}{\Omega}$$

- fhrp - diskretni

$$\mathcal{P}(X_i, n)$$

$$\sum_{\forall i} \mathcal{P}(X_i, n) = 1$$

$$\hat{\mathcal{P}}(X_i, n) = \frac{\text{count}(\xi_\omega[n] = X_i, n)}{\Omega}$$

- fhrp spojite

$$\mathcal{P}(x, n) = ???$$

$$p(x, n) = \frac{dF(x, n)}{dx}$$

$$v(t) = \frac{dl(t)}{dt} = \frac{l(t_2) - l(t_1)}{t_2 - t_1} = \frac{\Delta l}{\Delta t}$$

$$\rho(x, y, z) = \frac{dm}{dV} = \frac{m(x_1 \dots x_2, y_1 \dots y_2, z_1 \dots z_2)}{(x_2 - x_1)(y_2 - y_1)(z_2 - z_1)} = \frac{\Delta m}{\Delta V}$$

$$\text{histogram}(x \in \text{interval}, n) = \text{count}(\xi_\omega[n] \in \text{interval}, n)$$

$$\mathcal{P}(x \in \text{interval}, n) = \frac{\text{count}(\xi_\omega[n] \in \text{interval}, n)}{\Omega}$$

$$p(x \in \text{interval}, n) = \frac{\text{count}(\xi_\omega[n] \in \text{interval}, n)}{\Omega \Delta}$$

sumy do jedne ...

$$\int_t v(t) = ??$$

$$\int \int \int_V \rho(x, y, z) = ??$$

$$\int_{x=-\infty}^{+\infty} p(x, n) = 1$$

2d veci ...

$$\begin{aligned} \mathcal{P}(X_i, X_j, n_1, n_2) \\ p(x_i, x_j, n_1, n_2) \end{aligned}$$

$$\text{joint probability} = \frac{\text{count that something happened simultaneously in } n_1 \text{ AND } n_2}{\text{total}}$$

$$\hat{\mathcal{P}}(X_i, X_j, n_1, n_2) = \frac{\text{count}(\xi_\omega[n_1] = X_i \text{ AND } \xi_\omega[n_2] = X_j)}{\Omega}$$

$$\text{histogram}(x_1 \in \text{interval}_1, x_2 \in \text{interval}_2, n_1, n_2) = \text{count}(\xi_\omega[n_1] \in \text{interval}_1, n_1 \text{ AND } \xi_\omega[n_2] \in \text{interval}_2, n_2)$$

$$\mathcal{P}(x_1 \in \text{interval}_1, x_2 \in \text{interval}_2, n_1, n_2) = \frac{\text{count}(\xi_\omega[n_1] \in \text{interval}_1, n_1 \text{ AND } \xi_\omega[n_2] \in \text{interval}_2, n_2)}{\Omega}$$

$$p(x_1 \in \text{interval}_1, x_2 \in \text{interval}_2, n_1, n_2) = \frac{\text{count}(\xi_\omega[n_1] \in \text{interval}_1, n_1 \text{ AND } \xi_\omega[n_2] \in \text{interval}_2, n_2)}{\Omega \Delta^2}$$

momenty - ocekavani

$$a[n] = E\{\xi[n]\}$$

expectation = sum over all possible values what we're expecting

$$a[n] = \sum_{\forall X_i} \mathcal{P}(X_i, n) X_i$$

$$a[n] = \int_x p(x, n) x dx$$

rozptyl

$$D[n] = E\{(\xi[n] - a[n])^2\}$$

$$D[n] = \sum_{\forall X_i} \mathcal{P}(X_i, n) (X_i - a[n])^2$$

$$D[n] = \int_x p(x, n) (x - a[n])^2 dx$$

prime souborove odhady ...

$$\hat{a}[n] = \frac{1}{\Omega} \sum_{\omega=1}^{\Omega} \xi_\omega[n]$$

$$\hat{D}[n] = \frac{1}{\Omega} \sum_{\omega=1}^{\Omega} (\xi_\omega[n] - \hat{a}[n])^2$$

correlation coeff

$$R[n_1, n_2] = E\{\xi[n_1] \xi[n_2]\}$$

$$R[n_1, n_2] = \sum_{\forall X_1} \sum_{\forall X_2} \mathcal{P}(X_1, X_2, n_1, n_2) X_1 X_2$$

$$R[n_1, n_2] = \int_{x_1} \int_{x_2} p(x_1, x_2, n_1, n_2) x_1 x_2 dx_1 dx_2$$

$$\hat{R}[n_1, n_2] = \frac{1}{\Omega} \sum_{\omega=1}^{\Omega} \xi_\omega[n_1] \xi_\omega[n_2]$$

stacio

$$F(x, n) \rightarrow F(x) \quad p(x, n) \rightarrow p(x)$$

$$a[n] \rightarrow a \quad D[n] \rightarrow D \quad \sigma[n] \rightarrow \sigma$$

$$p(x_1, x_2, n_1, n_2) \rightarrow p(x_1, x_2, k)$$

$$R[n_1, n_2] \rightarrow R(k)$$

ergo tady pak skrtnout to s omegou ...

$$\xi_\omega[n] \Rightarrow \xi[n]$$

$$\hat{a} = \frac{1}{N} \sum_{n=0}^{N-1} \xi[n] \quad \hat{D} = \frac{1}{N} \sum_{n=0}^{N-1} [\xi[n] - \hat{a}]^2 \quad \hat{\sigma} = \sqrt{\hat{D}}$$

$$\hat{R}[k] = \frac{1}{N} \sum_{n=0}^{N-1} \xi[n] \xi[n+k]$$

$$\hat{\mathcal{P}}(X_i, X_j, k) = \frac{\text{count}(\xi[n] = X_1 \text{ AND } \xi[n+k] = X_2)}{N}$$

———— R[k] —————

$$R[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n]x[n+k],$$

$$R_{xy}[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n]y[n+k],$$

$$R[k] = \frac{1}{N-|k|} \sum_{n=0}^{N-1} x[n]x[n+k],$$

———— power spec —————

$$G(e^{j\omega}) = \sum_{k=-\infty}^{\infty} R[k]e^{-j\omega k}$$

$$R[k] = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\omega})e^{+j\omega k} d\omega$$

$$G(e^{j\omega}) = G(e^{-j\omega})$$

$$G(e^{2\pi \frac{k}{N}}) = DFT\{R[n]\}$$

$$G(e^{2\pi \frac{k}{N}}) = \frac{|DFT\{x[n]\}|^2}{N}$$

———— powers —————

$$P = \frac{1}{N} \sum_0^{N-1} x^2[n]$$

$$R[0] = \frac{1}{N} \sum_{n=0}^{N-1} x[n]y[n+0] = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

$$D = \frac{1}{N} \sum_{n=0}^{N-1} (x[n] - a)^2 = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

$$P = D + a^2$$

$$P = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\omega})d\omega$$

$$R[0] = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\omega})e^{+j\omega 0} d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\omega})d\omega$$

$$P_{\omega_1, \omega_2} = 2 \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} G(e^{j\omega})d\omega = \frac{1}{\pi} \int_{\omega_1}^{\omega_2} G(e^{j\omega})d\omega$$

———— white noise —————

$$G(e^{j\omega}) = \sum_{k=-\infty}^{\infty} R[k]e^{-j\omega k} = R[0]e^{-j\omega 0} = R[0]$$

$$p(x) = \begin{cases} \frac{1}{\Delta} & \text{for } |x| < \frac{\Delta}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$p(x) = \mathcal{N}(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{|x-\mu|^2}{2\sigma^2}}$$

———— filtering —————

$$G_y(e^{j\omega}) = |H(e^{j\omega})|^2 G_x(e^{j\omega})$$

———— SNR —————

$$SNR = 10 \log_{10} \frac{P_x}{P_e} \quad [\text{dB}].$$

$$SNR = 10 \log_{10} \frac{\frac{1}{N} \sum_{n=0}^{N-1} x^2[n]}{\frac{1}{N} \sum_{n=0}^{N-1} e^2[n]} = 10 \log_{10} \frac{\sum_{n=0}^{N-1} x^2[n]}{\sum_{n=0}^{N-1} e^2[n]}$$

quant

$$P_e = D_e = \int_{-\infty}^{\infty} g^2 p_e(g) dg = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} g^2 \frac{1}{\Delta} dg = \frac{1}{\Delta} \left[ \frac{g^3}{3} \right]_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \frac{1}{3\Delta} \left( \frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right) = \frac{\Delta^2}{12}$$

$$\Delta = \frac{2A}{L} \quad P_e = \frac{\Delta^2}{12} = \frac{4A^2}{12L^2} = \frac{A^2}{3L^2}.$$

$$SNR = 10 \log_{10} \frac{P_x}{P_e} = 10 \log_{10} \frac{\frac{A^2}{2}}{\frac{A^2}{3L^2}} = 10 \log_{10} \frac{3L^2}{2}.$$

$$SNR = 10 \log_{10} \frac{3}{2} (2^b)^2 = 10 \log_{10} \frac{3}{2} + 10 \log_{10} 2^{2b} = 1.76 + 20 b \log_{10} 2 = 1.76 + 6b \quad [\text{dB}].$$