

VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

BRNO UNIVERSITY OF TECHNOLOGY

FAKULTA INFORMAČNÍCH TECHNOLOGIÍ
ÚSTAV INTELIGENTNÍCH SYSTÉMŮ

FACULTY OF INFORMATION TECHNOLOGY
DEPARTMENT OF INTELLIGENT SYSTEMS

INTELLIGENT MOBILE ROBOT

DISERTAČNÍ PRÁCE

PHD THESIS

AUTOR PRÁCE

AUTHOR

RADIM LUŽA

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Abstrakt

Tato práce se zabývá řešením lokalizace v rozsáhlých venkovních prostředích s řídkými orientačními body. Důraz je kladen na nezávislost na externích systémech. Práce popisuje stávající metody a algoritmy používané pro lokalizaci. Dále je popsán návrh a realizace řešení pro lokalizaci v rozsáhlých venkovních prostředích. Implementované řešení bylo otestováno sadou experimentů. Na závěr jsou výsledky experimentů konfrontovány s cíli práce a jsou navrženy možnosti dalšího vývoje.

Abstract

The thesis concerns about solution for localization of mobile robot in large outdoor environment with sparse landmarks. The emphasis is on independence on external system. The thesis describes existing localization methods and algorithms used for localization. Then the solution for localization in large outdoor areas is designed and implemented. The implemented solution is tested by set of experiments described further in the thesis. Finally the conclusion is made - results are confronted with goals of the thesis and also ways of future development are projected.

Klíčová slova

Robotika, lokalizace ve venkovním prostředí, feature-based mapování.

Keywords

Robotics, outdoor localization, feature-based mapping.

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Intelligent mobile robot

Prohlášení

Prohlašuji, že jsem tuto práci vypracoval samostatně pod vedením doc. Ing. Františka V. Zbořila CSc. Uvedl jsem všechny literární prameny, ze kterých jsem čerpal.

.....

Radim Luža
March 12, 2019

Poděkování

Děkuji svému školiteli Františkovi V. Zbořilovi za ochotné vedení při tvorbě práce a své rodině za podporu.

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Tato práce vznikla jako školní dílo na Vysokém učení technickém v Brně, Fakultě informačních technologií. Práce je chráněna autorským zákonem a její užití bez udělení oprávnění autorem je nezákonné, s výjimkou zákonem definovaných případů.

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Chapter 1

Introduction

In today's world the robots are becoming more and more frequent tool for solving tasks from various areas of human effort. In some areas like manufacturing they already have a tradition while in other the usage of the robots is still in experimental phase. With growing performance and decreasing energy consumption of computers the complexity and performance of robots control systems grow rapidly. This allows robotic researches and developers to apply a more sophisticated attitudes and use more advanced algorithms for controlling the robots. Thus the robots can operate in more complicated environments and solve more complex tasks. In general robots are becoming more autonomous and more helpful.

First autonomous robots appeared in industrial plants and depots where they operated in well defined conditions and well known environment. Using robots in industry was a great success and it was a matter of time when the robots will spread into other areas. As the technology advances the robots can deal with uncertainty, changing conditions and non-deterministic environments. One of the most complicated environments for controlling the robots is outdoor environment. The environment has many variants according to particular place on the planet. Moreover it is often dynamic with both slow and fast changes happening in real time. Thus it is not easy to make generic assumptions about the outdoor environment. This fact complicates design of robots for outdoor environment in terms of both hardware and software.

For most of the tasks the robots accomplish it is essential to receive feedback for executed actions. Robot needs to know how the environment reacts on its actions and in case of mobile robots where in the environment the robot is at the particular time. The problem of estimating actual location of the robot is called localization. Despite the latest advances in robotics the localization in the outdoor environment is still a challenge. There is probably no generic solution that would fit all purposes. It is usually necessary to adapt localization to particular use case. There are several support systems for outdoor localization. Probably the most frequently used is network of satellites - so called GNSS (Global Navigation Satellite System). Thanks to signal from satellites it is possible to localize the mobile robot with very good precision. Unfortunately there are situations in which it is not possible to use these satellite networks.

Creating of generic localization solution without support of external systems is not easy. The environment usually has to be restricted to particular variant or set of variants. This thesis concerns about solution for localization in large outdoor areas like meadows, fields or airport runways. Typical aspect of these environments is that there are only few objects that can be used as landmarks. The solution described in this thesis tries to deal with this

situation using sensors installed on the robot - particularly camera, laser rangefinder and odometry.

Chapter 2

Theoretical background of robot localization

Localization is one of the essential components of autonomous robot behaviour. Most of autonomous robot missions depend on the knowledge of robot's location relative to the mission goals and obstacles. This chapter deals with theoretical background for localization and SLAM (Synchronous Localization And Mapping) process.

2.1 Localization problem

The localization problem can be intuitively described as finding location of the robot in given environment or more precisely finding location of the robot relatively to the origin of given coordinate system. For stationary robots the localization of the endpoint can be solved by precise measurement of joint poses. Unfortunately this approach is insufficient in case of mobile robots due to slippage of undercarriage. Still the technique of measuring robot pose according to speed or change of pose of motivators is used as a one of location sources. In mobile robotics the data from the chassis are called odometry. Technique of estimating new position of the mobile robot according to known last position and relative change of position measured by sensors is called dead reckoning.

The localizaiton problem can be cathegorized according to several points of view:

Local vs global Local localization can be intuitively described as correcting prior pose information or trajectory following. The prior pose information with limited error is essential for local localization. The robot pose uncertainty can be approximated using unimodal distribution - usually Gaussian. The global localization on the other hand can not rely on any limit of pose error in prior pose information. For many global localization algorithms the initial pose is chosen randomly. It is obvious that the global localization problem is more complicated than local localization problem. Another cathegory of localization problem is introduced in [35] - the kidnapped robot problem. The word „kidnapped“ in this context means that the robot was moved to another pose without knowing that. It doesn't reflect a real world situation but algorithms that can deal with the kidnapped robot problem can also deal with situations when robot gets lost (for example due to insufficient interesting objects in its surrounding).

Static environment vs dynamic environment In the static environment the only object that change its pose over time is the robot. All other objects keep their poses

and also their features and robot observing the environment can rely on it. The static environment is typical for simulators. Most of the real world environments are dynamic. In dynamic environments the features of objects including shape (for example tree in wind), pose (for example open vs closed door) and surface color (due to changing light conditions) change with the time. Despite the real world environments are almost always dynamic the static algorithms work there too - small changes in the environment can be considered as errors and they can be filtered out.

Passive vs active In passive approach the localization algorithm works as a passive observer - it takes data from sensors and tries to estimate location of a robot that is controlled by another entity. In active approaches the localization algorithm also controls robot motion to adapt its trajectory for needs of localization. The trajectory adaptation is affected by sensor capabilities and structure of the environment (for example robot stays close to the walls and obstacles because of limited reach of sensors).

2.2 Sensors

In today's robotics there are many sensors usable for purposes of localization of the robot and for creating maps of surrounding environment. Listing all of these sensors is out of scope of this thesis. Existing sensors can be categorized by principle of operation. We can define following categories of sensors:

Range finder Range finder in general is a device that can measure distance to obstacle. There are several principles of ranging used in today's robotics that differ in its properties. Probably the cheapest range finders are based on ultrasonic principles. These are the only range finders that do not use electromagnetic waves - they use mechanical waves instead. Ultrasonic range finder generates a short density change in the air usually by pulsing membrane. This change spreads through the medium - usually air or water as advancing mechanical wave. When the wave hits obstacle it is reflected and travels back to range finder that is already listening. The delay between sending the wave and receiving it is directly related to distance the wave had to travel. The relation is defined in equation 2.1. The distance has to be divided by 2 as the wave travels from the range finder to the obstacle and back.

$$d = \frac{\Delta t \cdot c}{2} \tag{2.1}$$

Speed of the mechanical wave in a gas c including air varies according to density of the media. The relation is described by Newton-Laplace equation 2.2 cited from [24]. The K_s is so called elastic bulk modulus that is defined as multiplication of pressure in the media and temperature of the media. ρ is density of the media. Speed of mechanical wave increases with growing pressure and decreases with growing density of the media. One important attribute of ultrasonic range finder comes from this equation: Its calibration is valid only for particular temperature and pressure condition in particular media. With changing conditions the precision of the range finder drops.

Another attribute is rather wide cone in which the ultrasonic range finder detects obstacle. The range finder can not find exact position of the object. It can only find

object lying somewhere on equidistant spherical surface. Of course this applies also to any beam but in case of ultrasonic rangefinder the cone is too wide to be neglected during finding exact position of particular object. Range of ultrasonic rangefinders varies from tens of centimeters to tens of meters. The range is limited by acoustic power of the transmitter and also by a lot of noise generated by other mechanical waves (sounds) in the media.

$$c = \sqrt{\frac{K_s}{\rho}} \quad (2.2)$$

Radar ranging is another principle used for measuring distances. It works in a very similar way to ultrasonic rangefinder but it differs in physical principle of the transmitted wave. Radar rangefinder use electromagnetic radio wave instead of mechanical wave. Again the wave reflects from obstacle and it is received again by the radar sensor to measure time difference. As the wave is electromagnetic it is immune to mechanical waves on one hand but it interferes with electromagnetic waves on the same or similar frequencies. As the attenuation of electromagnetic wave in the air is smaller than attenuation of mechanical wave the range of the radar sensors can be higher reaching hundreds of meters. High performance radar system can reach tens of kilometers far.

Laser rangefinders use electromagnetic wave as radar rangefinder does but it uses it in a different way. As first - it uses light instead of radio - the frequency of electromagnetic waves differ a lot. And secondly the wave is focused into narrow beam. The beam is so narrow that in most of applications it can be considered as a single point. This is one of the most significant differences in abstraction of laser rangefinder and other rangefinders described above. Reach of laser rangefinder can vary from tens of meters up to kilometers. To measure time that light needs to reach the obstacle, reflect and arrive back a more sophisticated attitude is needed as the delay is very small and speed of light is similar to speed of electrons in conductors. Naive approach of sending pulse and waiting for arrival of the reflection is not very usable. Instead a phase shift of modulated signal is used. The carrier wave that is transmitted is modulated with a known pseudorandom code. The receiver continuously receives the wave and reads the code. The distance is measured as a phase shift of the transmitted and received code sequences. The resolution is given by wave length and length of code symbols in wave periods. This attitude brings one shortage: The range of rangefinder is limited not only by transmitting power but also by length of code sequence. When the code sequence starts to repeat the rangefinder „sees a ghosts“ which means that it sees distant objects much closer to the rangefinder than they really are.

Single point ranging is often not sufficient for many applications. This is why 2D and 3D laser rangefinders were invented. Principle of these devices is based on rotating a single point laser rangefinder or rotating a precise mirror in front of static single point laser rangefinder. This way it is possible to obtain from tens to millions of distance measurements in one scanning period (usually a revolute of device head). Such a dense data can be used to construct precise maps and model surrounding of the robot.

The laser transmitter has very narrow spectrum - in theory a single wavelength only. This property can be used to avoid interference of several laser rangefinders but it

also allows to mix several frequencies in one scan. Each color of the spectrum has a different properties especially in terms of reflection. Particular color reflects the best from the surface of the same color. This can be used to obtain color information together with distance information but light of different frequencies differ also in penetration capabilities. Some frequencies can penetrate water or grass while other reflect from its surface. This way it is possible to obtain additional information about terraing and surroundings in a single, aligned scan. One of sensors offering such functionality can be found in [30].

Last in this cathegory are infrared rangefinders. There are two principles of measuring distance used in existing infrared rangefinders. First of them is time of flight principle - the same as the laser rangefinders use. Infrared light is also eletromagnetic signal so processing works in a very similar way. Another principle is to project matrix of infrared points to the surrounding and detect them with infrared camera. According to deformation of the matrix the distances of particular points in the 3D space can be compute. This principle described in [16] is used by Kinect 1 sensor. Advantage of infrared rangefinders over laser rangefinders are mainly manufacturing costs. Draw-back of infrared sensing is that it is significantly affected by daylight that contains also infrared component.

Cameras Cameras are sensors that provide data with a very high information density. Unfortunately the information is not structured and finding the required information in camera image is still important research topic. Basicaly the camera is a sensor that compounds of matrix of light sensitive cells and optical aparatus. The chip with matrix of light sensitive cells converts light intensity into electric signal according to conversion technology used. The signal is evaluated by camera control electronics and converted to digital value. For common cameras the value is usually 8 bits per channel. According to detected information the cameras can be distinguished to grayscale cameras (light intensity), color RGB cameras (intensity of red, green and blue channel) and special cameras (for example thermocamera that receives infrared electromagnetic signal).

Depth cameras Cathegory of depth cameras cover cameras extended with depth information for every pixel of the image.

Stereocamera Stereocamera is a sensor composed of two RGB or grayscale cameras that work in a similar way as pair of eyes works. In the same scene observed by two cameras that are slightly shifted one from another the objects are slightly shifted too in camera images. Objects that are closer to the pair of cameras have larger shift than objects that are more distant. The shift is called disparity. In the best case we can define disparity for almost each pixel in the image. Knowing the disparity and camera parameters we can compute distance of given point from plane given by camera centers and normal parallel with camera axes. If we recognize pair of corresponding pixels in left and right image we can compute the disparity using eqaution 2.3. In the equation the x_l and x_r are coordinates of corresponing pixels in left and right image and x_{cl} and x_{cr} are horizontal center points of images. The f is focal length of both cameras (we suppose that the cameras have the same parameters) and B is baseline - the distance between

camera centers.

$$d = (x_l - x_{cl}) - (x_r - x_{cr}) \quad (2.3)$$

Knowing the disparity we can reconstruct $\langle x, y, z \rangle$ coordinates of the point in observed scene using following set of equations. For the equations to be valid there are some presumptions about the cameras. First the cameras has identical parameters. Fortunately todays manufacturing methods allow us to get cameras that are practically identical in point of view of camera model. Second presumption is that the camera images are perfectly rectified. The equations above do not count with deformation of image due to camera optics characteristics. The rectification is usually achieved with software postprocessing of camera image. Correct values for postprocessing are obtained by calibration process of cameras.

$$z = \frac{fB}{d} \quad (2.4)$$

$$x = \frac{(u - u_c)z}{f} \quad (2.5)$$

$$y = \frac{(v - v_c)z}{f} \quad (2.6)$$

Limitation of stereocamera performance is usually given by methods of detecting corresponding points in both images. In real situations only minority of pixels can be detected properly. Still the image can be segmented into regions with the same depth so the scene can be reconstructed with good precision. More details can be found in [13].

RGB camera with matrix depth sensor The matrix depth sensor can be based on several technologies. Probably the most frequently used are pattern projection and time of flight measurement. The pattern projection [16] is a method of projecting known pattern to the scene and compute 3D profile of the scene according to deformation of the pattern. The projection and observation of the pattern usually happens outside the visible spectrum to avoid noise in the RGB camera view. This method is used in first generation of Kinect sensor. Limitation of the sensor is given by limited output power of pattern projector that can reach up to tens of meters and also small resitance to light noise. In case of Kinect the sunlight usually makes the patter completely unreadable and even in indoor environment the daylight or artificial light significantly affect stability of depth data.

Another approach is measuring the time of flight of modulated light beam [21]. The modulated beam si projected to the scene and received back with receiver. According to phase shift of the projected beam in particular point the distance is computed. This approach is more robust in changing light conditions but it still has its limitations. Except of output power of the beam projector there is interesting problem when measurement exceeds the maximal distance the sensor was designed for. If the object reflecting the beam is so far that the time of flight is longer than the period of the transmitted signal then the sensor might observe „ghost“ measurements. The ghost measurement appears to be very close despite the real object is much further.

Encoders Encoders in general are sensors that provide information about position of moving components using digital code.

Optical Optical encoders [19] compound of three components: Light emitter, light receiver and moving part with sequence of contrast areas. The contrasting sequence can be achieved by alternating areas with high and low light reflection or by alternating areas with high and low opacity. In the first case the the light emitter and receiver aim in the same direction and receiver receives reflected light and in the second case the emitter heads against the receiver and beam of light is blocked by opaque areas. Optical encoders are convenient for measuring position of moving components because the output depends on actual intensity of reflected or passing light beam. Light beam is resistant against electromagnetic noise and vibration so position feedback provided by optical encoder can be very stable and reliable. Beam of light can be very narrow so it is possible to combine several one-bit encoders into more complex encoders. For motor shafts the Gray code AB phase encoder is typically used. Advantage of AB phase encoder is possibility to determine direction of rotation next to revolute speed. For manipulators with rotary joints and for other rotary joints with limited range of rotation the N-bit binary encoder is often used to provide information about absolute joint position. For purposes of localization the AB phase encoders are typically used for odometry and N-bit encoders are typically used as a feedback for sensor positioning.

Hall probes Hall probes use effect of force interaction of magnetic field on charged particles moving through it called Hall effect [5]. When beam of charged electrons flows through semiconductor and the semiconductor gets into magnetic field the flowing electrons are affected by magnetic field. Electron trajectory is deviated by magnetic field. If the magnetic field is orthogonal to electron beam the effect is maximized. As a consequence of electron beam deviation one plane of the semiconductor is charged positively due to lack of electrons and due to it the opposite plane gets negative charge. This charge creates a barrier in the semiconductor that flowing electrons have to overcome. The barrier appears as Hall voltage between semiconductor planes. Of moving part contains small magnets we can detect the moment when magnet moves around the hall sensor by change of the voltage. Output of hall sensors used as encoders are usually filtered by threshold circuits that forms rectangular output of the encoder.

Hall sensors are usually used as encoders on motor shafts on AC motors and other rotary components. Hall probes are convenient especially for speed regulation as they react especially on change in magnetic field and they have short reaction times.

Electromagnetic Electromagnetic encoders use effect of electromagnetic induction to detect position of moving part. There are many approaches [40] using different encoding area structure with different windings. Particular designs have its cons and pros. Basically these encoders are used mainly for measuring speed of motion and phase machine period but they can be also used for absolute position measurement.

IMU IMU is acronym for Inertial Measurement Unit - a composite sensor intended for measuring orientation and acceleration of the moving object in 3D space [17]. The

IMU typically compounds of three accelerometers, three gyroscopes and sometimes additionally of three magnetometers. By integrating acceleration vector in time while knowing orientation of the moving object we can compute its velocity and position in time. This is used for estimating pose of the mobile robot during its operation. Process of estimating position of the robot from IMU and odometry is called dead-reckoning. The dead-reckoning is used as one of robot pose information sources for localization and SLAM algorithms. The most important advantage of dead-reckoning is that it does not rely on changing surrounding environment. It relies mostly on gravity and magnetic forces of the planet to estimate where did the robot get from the initial position. A serious disadvantage of dead-reckoning is error accumulation. Every measurement is affected by error. No matter how large the error is every upcoming measurement brings additional error to pose estimation. Due to this the dead-reckoning loses precision over the time. When the dead-reckoning is fused with other localization approaches using external data from the environment the growing error can be „reset“ to low value from time to time. In such mode of operation the dead-reckoning provides very useful source of localization data. Even in the cases when dead-reckoning error is not reset by any other localization system it provides at least some idea of location of the robot.

GNSS GNSS is acronym for global navigation satellite system. This generic acronym covers navigation systems that use network of satellites orbiting the Earth to measure exact position of rover on planet surface. In today's world there are three most important satellite navigation systems: GPS, GLONASS and GALILEO [7]. GPS is controlled by USA, GLONASS by Russian federation and GALILEO by European union.

GNSS in general are complex systems compounding of satellites on Earth's orbit, ground control and rover receivers. Describing each system in the detail is out of scope of this thesis so only GPS will be briefly described. Other GNSS use similar concepts. As described in [8] position measurement using GPS is based on measuring distances of the rover from GPS satellites. GPS satellites and also ground rover have code generator that generates pseudorandom code from category of Golden codes. In GPS each satellite has its own seed for pseudorandom code generator so generated sequences are unique for each satellite. The rover generates code sequences for each satellite it observes. As the rover is receiving signal from a satellite the signal is modulated by the code sequence unique for the satellite. The rover compares the sequence with the one it generated itself. As the time is synchronized between rover and the satellite the rover can compute time shift between received and generated code. The timeshift has a linear dependency with the distance between rover and the satellite. The distance is radius of sphere about the satellite on which the rover lies. As rover knows the position of each satellite it can compute its own position at intersection of spheres. To get unambiguous position the rover needs to observe three or more satellites. With two satellites the rover can at least estimate its position on intersecting curve of two spheres given the model of Earth geoid but this estimate can be ambiguous.

Precision of position estimation is limited by code baudrate and it is also affected by atmospheric disturbance, effect of refraction of the signal when entering the more dense atmosphere and also by multipath signal spreading. Especially the atmospheric disturbances and multipath signal spreading bring random noise to the position mea-

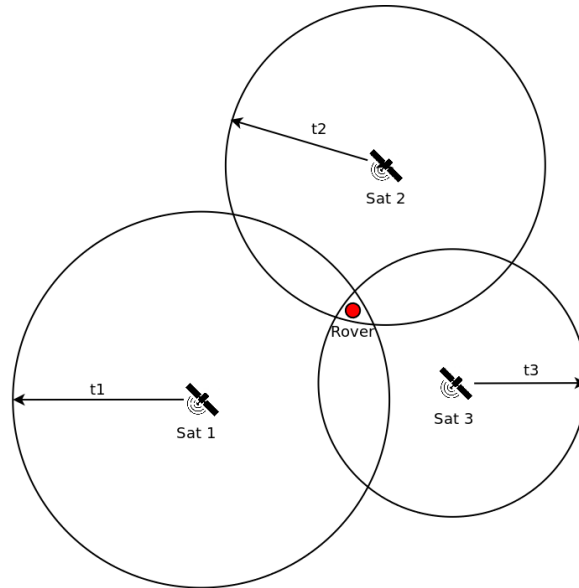


Figure 2.1: GPS principle: geometry.

surement. This effect can be eliminated by measuring position on a spot for a longer period of time but this is not an option for moving clients like vehicles or mobile robots. The effect of random noise can be significantly reduced for mobile clients using differential GPS (DGPS) [25]. In this approach the GPS system is extended with base station that is nearby the rover (up to couple of kilometers usually) that measures its GPS position for a long period of time and integrates noisy measurements. The integration eliminates random errors and the station gets its precise position. During mobile rover operation the base station receives its GPS position in real time and compares it with its known position. The difference is sent to the rover that subtracts the difference from the position it measures and gets better estimation of its real position. The system works with the presumption that same effect that brings error to GPS position of the station affects also the GPS position of the rover.

Precision of GPS can be increased measuring phase shift of carrier signal instead of the code period. This allows to improve precision of GPS to centimeters.

As the GPS is today's most frequent way of outdoor localization a basic mathematical background is described in following paragraphs. From mathematical point of view the GPS localization is based on N-lateration where $N \geq 3$ as described in [36]. As mentioned before satellites travel around the planet elliptic trajectories - the orbits. Trajectories of satellites are known and so is known the exact position of satellites on the trajectories. When message from satellite is received it contains a timestamp. According to this timestamp it is possible to measure time difference between satellite and rover so it is possible for rover to compute the distance between rover and the satellite. The distance defines sphere about the satellite on which the rover's actual position lies. When distance to another satellite is measured position of the rover is restricted to circle defined by intersection of two spheres. In theory once the rover obtains third distance the exact position of the rover can be computed as depicted in figure 2.1.

The figure shows the situation with three satellites observed by rover. Time that takes

the signal to arrive from each satellite is denote by t_1 , t_2 and t_3 respectively. The particular time difference measured as these shift of pseudorandom code generated by the satellite makes a spherical surface about the satellite. We are trying to find the particular time differences - diameters of spheres, where all three spheres intersect at rover's position.

From mathematical point of view we are finding solution of set of equations 2.7. Each of the equations describe one spherical surface about one of satellites. Identification of the satellite is given by index. All equations together define the trilateration principle: We are finding intersection point of spherical surfaces. As we can observe in the equation set 2.7 there are four equation instead of three. The reason is that we need to find four unknown variables: x_r , y_r , z_r and ϵ . The three variables define position of the rover on planet surface. The ϵ denotes a time difference of rover's clock. This another variable was added due to technical reasons. Despite the satellites have very precise time synchronization it is not easy to achieve such a precise synchronization for rover's clock - it would make GPS receiver very expensive and clock synchronization for rovers would require additional infrastructure. To avoid these technical limitations the fourth variable epsilon was added.

Function $d(t_x, \epsilon)$ is function of distance according to time difference of signal travelling from satellite to rover and difference of rover's clock. In optimal case the fourth distance is obtained by fourth satellite observation but if the satellite is not available it is possible to use three satellites only and use geometry model of Earth as the fourth distance. In this simple model the Earth is modelled as sphere. This model can be used for initial estimation without any prior knowledge. After approximate coordinates are estimated the precision can be improved using proper Earth geoid model [14]. Still this is more like a fallback solution that is less precise than observing fourth satellite.

$$\begin{aligned}
(x_r - X_1)^2 + (y_r - Y_1)^2 + (z_r - Z_1)^2 &= d(t_1, \epsilon) \\
(x_r - X_2)^2 + (y_r - Y_2)^2 + (z_r - Z_2)^2 &= d(t_2, \epsilon) \\
(x_r - X_3)^2 + (y_r - Y_3)^2 + (z_r - Z_3)^2 &= d(t_3, \epsilon) \\
(x_r - X')^2 + (y_r - Y')^2 + (z_r - Z')^2 &= d(t', \epsilon)
\end{aligned} \tag{2.7}$$

With additional equation the required precise informations that rover needs to know to make GPS localization work are: a) Observation of three satellites, b) Exact trajectory of satellites, c) Position of the satellite on the trajectory and d) Clock with precise ticks.

Clock need to measure time difference precisely but it does not need to be synchronized with satellites. Trajectories and position on the trajectory allows rover to compute exact position of the satellite in moment when it set signal to the rover. This way the coordinates X_N, Y_N, Z_N are being obtained. To obtain information about GPS satellites the data encoded in the GPS signal contain almanac - the overview information about entire GPS system including trajectories (orbits) of the satellites and status of the satellites. Almanac also helps receiver to estimate which satellites could be visible according to its last known location so it speeds up search for satellites. To obtain exact position fixes for particular satellites the signal of each satellite contains ephemeris - exact spherical polar coordinates of satellite at given time.

We can notice that in figure 2.1 the circles denoting the spherical surfaces do not

intersect perfectly. This imprecision is caused mostly by noise. There are several sources of the noise. Positions of satellites may not be perfectly precise, clock ticks of rover may be slower or faster than satellite's one but most of the noise is caused by atmospheric disorders. The density of Earth's atmosphere slows down the signal from the satellites. Unfortunately the rover has no information about actual density changes in the atmosphere and these changes are too random to be estimated. Error caused by atmospheric noise has normal distribution so exact position of static receiver can be obtained with good precision by averaging of measurements over long period of time. This is not usable for mobile rovers of course but static receiver with known precise position can with localization of rovers nearby. If the rover is nearby the static receiver (up to tens of kilometers) the error of the rover will be almost the same as error of the static receiver. Static receiver can measure the error, compute correction and provide to rover so the rover can correct its position measurements. This is the way how Differential GPS that is mentioned above works.

Chapter 3

Synchronous localization and mapping

Previous chapter concerned about localization of mobile robot in previously known map. In real scenarios it is very typical that there is no map known before the robot is launched and its up to it to create one. Process of creating the map and self-localizing in it in the same time is called SLAM (acronym for Synchronous Localization and Mapping). More formally the SLAM problem can be described as finding map, actual location of the robot and trajectory from initial pose while we know the the initial pose and trajectory of control commands for the robot since initial time to actual time t . The SLAM problem is defined in 3.1 taken from [37].

$$p(x_t, m | z_{1:t}, u_{1:t}) \tag{3.1}$$

We can distinguish between on-line SLAM where the only last pose of the robot is being estimated while the trajectory remains unchanged and full SLAM when we update also the trajectory heading to actual pose of the robot. For most of applications finding the map and location in it is sufficient as it is important where the robot is rather than how it got to this place. If we assume that the prior poses of robot are known it also simplifies the SLAM problem itself. Instead of finding trajectories we are finding actual poses only according to prior pose, control and observations. This simplification bring a significant advantage: Map and actual pose are independent. We are still finding best map in which the robot pose estimation fits best but now we are optimizing expression 3.2.

$$p(x_t | z_{1:t}, u_{1:t}) \cdot p(m | z_{1:t}, x_{1:t}) \tag{3.2}$$

Advantage of this approach is that the complexity does not grow with length of trajectory traveled by robot. Of course the limitation is that if some incorrect estimation was done in the past it can not be corrected so the trajectory will be always incorrect. On the other hand in most of applications the trajectory is not needed so this disadvantage is not serious and it does not affect quality of new pose estimation.

3.1 Kalman filter based SLAM

Kalman filter based SLAM algorithms use Kalman filter for estimation of robot pose according to motion prediction and observations in time t . For localization of mobile robot

in unknown environment the extended kalman filter needs to be used as most motion of observation models are not linear. Replacing non-linear model with local jacobian allows us to use Kalman filter algorithm as defined by set of equations ?? anyway. The only difference is that the in prediction step the prediction of new robot pose according to control is not a simple linear motion but it uses generic non-linear but continuous function to predict new pose of the robot $\bar{\mu}_t$. Estimation of new covariance matrix uses jacobian of motion model instead of linear multiplication. A similar modification applies for observation model that is replaces by non-linear function $h(\mu(t))$ and its jacobian H_t . Entire algorithm of extended Kalman filter after modification can be observed in 3.3.

$$\begin{aligned}
\bar{\mu}_t &= g(u_t, \mu_{t-1}) \\
\bar{\Sigma}_t &= G_t \Sigma_{t-1} G_t^T + R_t \\
\bar{K}_t &= \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1} \\
\mu_t &= \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t)) \\
\Sigma_t &= (I - K_t H_t) \bar{\Sigma}_t
\end{aligned} \tag{3.3}$$

In Kalman filter base SLAM algorithms the entire state is represented by one highly-dimensional gaussian. The state contains pose of the robot, pose of landmarks, covariance of robot pose, covariances of landmark observations and also covariances between robot pose estimates and landmark pose estimates. The state vector is described by equation 3.4 and covariance matrix is described by 3.5.

$$x_t = [\mu_R, \mu_{l1} \cdots \mu_{lN}] \tag{3.4}$$

$$\Sigma = \begin{bmatrix} \Sigma_{R,R} & \Sigma_{R,l1} & \cdots & \Sigma_{R,lN} \\ \Sigma_{l1,R} & \Sigma_{l1,l1} & \cdots & \Sigma_{l1,lN} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{lN,R} & \Sigma_{lN,l1} & \cdots & \Sigma_{lN,lN} \end{bmatrix} \tag{3.5}$$

Initially the algorithm starts with $x_t = \mu_t$ and $\Sigma_t = \Sigma_{R,R}$. With any new observation the state vector and covariance matrix is extended with pose and covariance matrixes of the new landmark. If the landmark is re-observed its pose is updated together with update of robot's pose and of course covariance submatrixes connected with the landmark are updated too. Kalman filter based SLAM algorithms are relatively straightforward application of Kalman filter to SLAM problem but there are several limitations that limit usage of this family of algorithms.

The most important aspect of Kalman filter based SLAM algorithms is computational complexity. Entire state of the SLAM including robot pose and map is represented by a single state vector and single covariance matrix. As we can observe every new landmark extends the state vector by M dimensions and covariance matrix by M dimensions in each direction. For example for 3D SLAM with robot pose represented by 6D vector and landmark pose represented by 3D vector with corresponding 6x6 and 3x3 covariance matrixes respectively the size of state vector will be $6 + 3N$ and dimensions of covariance matrix will be $\langle 6 + 3N, 6 + 3N \rangle$. Asymptotic computational complexity of the algorithm is $\mathcal{O}(n^2)$ where n is amount of observed landmarks ($n = N$). The complexity is given by operations with covariance matrix.

Another limitation of Kalman filter based SLAM algorithms is requirement for correct landmark association. It means that once the landmark is re-observed we need to ensure that we recognize that landmark as the proper one that was already observed. If the association fails the state will be extended with a „ghost“ landmark that will increase complexity. Moreover that false detected landmark may compete with previously observed landmark for association and thus could bring even more uncertainty instead of improving estimations. As the state of system represent only one hypothesis with uncertainty there is no recovery after this hypothesis becomes wrong.

3.2 Particle filter based SLAM

Particle filter is non-parametric recursive Bayes filter. In particle filter based SLAM methods the posterior is represented by set of particles so unlike the Kalman filter based algorithms the posterior distribution is not limited to gaussian. The abstract steps of the algorithm are very similar to Kalman filter. First new proposal distribution is predicted according to actual pose and control. The proposal is represented by set of particles generated from actual pose of the robot x_t using motion model with uncertainty. Generating new set of particles is actually sampling of the proposal distribution as drafted in equation 3.6.

$$x_t^{[i]} \sim proposal(x_t|x_{t-1}, u_t) \quad (3.6)$$

In next step particle weights are updated. In this process the observation model is utilized to convert measurements to observations. Each particle is evaluated with update of its weight. The weight update reflects how well the particle fits into target distribution given by observations. A generic weight update equation is 3.7.

$$w_t^{[i]} = \frac{target(x_t^{[i]})}{proposal(x_t^{[i]})} \quad (3.7)$$

Finally the particles are re-sampled according to its weights to choose the best particles for next generation. This generic algorithm is the same as generic particle filter algorithm described in chapter ???. The importance of particle filter for SLAM is semantics of particle. Each particle represents one hypothesis about pose of the robot in the environment and about model of the environment (map) - shortly the particle represents hypothesis about actual state. In case of feature-based SLAM algorithms that represent map as a set of features the state representation (particle) contain also set of these features.

Important aspect of particle filter based SLAM algorithms is that they maintain a set of various hypotheses instead of single hypothesis like Kalman filter based algorithms. Moreover new hypotheses are generated in every iteration during proposal sampling. This approach allows to correct improper hypotheses containing for example broken maps or very unlikely pose estimation. Also the landmark association process itself can be simplified with set of particles representing various association hypotheses. By the time the incorrect hypothesis with high probability die out while the correct one will survive.

3.3 FAST-SLAM

The basic FAST-SLAM algorithm is a particle filter based SLAM algorithm with feature-based map representation. Every object in the map is represented by vector of features.

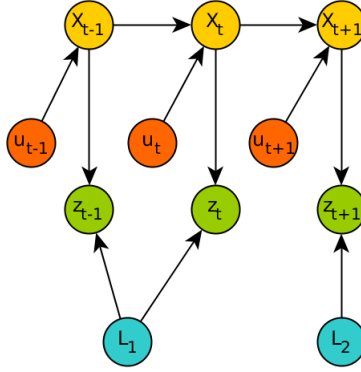


Figure 3.1: Conditional dependency of observation in SLAM process.

Problem of particle filter approach to modelling uncertainties is a high dimensionality. Count of dimensions grows linearly with count of represented objects in the map. If we represent thousands of objects we get to thousands of dimensions we need to represent. To cover such a high dimensional space we need a very high amount of particles. Such a solution would be computational demanding.

In FAST SLAM every particle represents actual map as a vector of features of all mapped objects and trajectory of the robot from initialization to actual time as described as described in 3.8.

$$i_j = \langle x_{j,1:t}, m_{j,1}, m_{j,2} \dots m_{j,N} \rangle \quad (3.8)$$

Figure 3.1 shows conditional dependency of landmark L_x observations on robot new pose X_{t+1} and previous trajectory of the robot $X_{t,t-1}, \dots$. The dependency graph corresponds to particle representation described in 3.8. If we look at each particle as a map (set of observations) given the robot pose we can say that the particular observations are independent on each other - given the robot pose. Thanks to this presumptions we can decompose the particle representation into robot pose and set of independent N-dimensional observation coordinates. This gives us two important advantages: First - we need to sample only probability distribution of robot pose and second - we can represent each observation with independent model. This simplifies the problem a lot. The set of particles will cover only possible robot poses. Each particle will „carry“ map of the surrounding environment describing how would the environment look if the robot pose was the pose represented by the particle. And the map of the environment will be significantly simplified - we can represent each observation with a small N-dimensional extended Kalman filter where N is number of dimensions whe SLAM works with (usually 2 or 3 dimensions).

The conditional probability of observation z_{t+1} is given by equation 3.9. The probability of $X_{t+1:1}$ can be expressed be equation 3.10. The equation 3.10 applies recursively for each position of hte robot in the history since beginning the the SLAM process.

$$P(z_{t+1}|L_2, X_{t+1:1}) = \frac{P(z_{t+1}, L_2, X_{t+1:1})}{P(L_2) \cdot P(X_{t+1:1})} \quad (3.9)$$

$$P(X_{t+1:1}) = P(X_t + 1|X_{t:1}, u(t+1)) \cdot P(u_{t+1}) \cdot P X_{t:1} \quad (3.10)$$

If we declare position in each step as known the dependency graph reduces to isolated subgraphs as can be observed in figure 3.2.

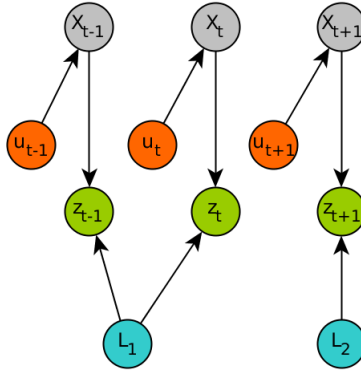


Figure 3.2: Conditional dependency of observation in SLAM process - given pose of the robot.

The conditional probability of z_{t+1} can be now expressed in much more simple way [3.11](#). This makes the situation much easier as there is no dependency on history. The dependency graphs [3.1](#) and [3.2](#) apply to motion model where we predict new pose of the robot according to last pose and control action. The dependency of the observation on the pose of the robot can be expressed in opposite direction as dependency of the robot pose on actual observation using Bayes formula [\[10\]](#). Saying that we know the pose of the robot in every point of its trajectory we can split complicated high-dimensional model of robot pose and all landmarks observed during robot travel into smaller independent models for each landmark.

$$P(z_{t+1}|L_2, X_{t+1}) = \frac{P(z_{t+1}, L_2, X_{t+1})}{P(L_2) \cdot P(X_{t+1})} \quad (3.11)$$

The process of reducing dimensionality without losing properties of probabilistic model is called Rao-Blackwellization. The Rao-Blackwellization is a more generic process of transforming estimator of unobservable random variable into estimator of observable random variable that satisfies the Rao-Blackwell theorem [\[3\]](#). In case of FAST SLAM we model joint distribution of robot trajectories and possible maps with conditional distribution of possible maps given robot trajectory multiplied with distribution of possible trajectories [\(3.12\)](#). Application of Rao-Blackwellization allows us to model the high-dimensional model compounding of robot trajectory and map by set of particles and apply particle filter on it.

$$P(x_{1:t}, m_{1:t}) = P(m_{1:t}|x_{1:t}) \cdot P(x_{1:t}) \quad (3.12)$$

The FAST SLAM algorithm compounds of four steps:

1. Prediction of new particle coordinates.
2. Update of particle weights according to observations.
3. Update EKF for observations.
4. Resampling - generating new set of particles.

3.3.1 Prediction of new particle coordinates

New particle coordinates are predicted using motion model of the robot. In general new particle pose of particles can be expressed by eq [3.13](#). The new particle is obtained by

sampling probabilistic model of new robot pose given prior pose of the particle x_{t-1} and control action u_t .

$$x_t^{[k]} \sim p(x_t | x_{t-1}^{[k]}, u_t) \quad (3.13)$$

3.3.2 Update of particle weights according to observation

New particle weight is updated using gaussian distribution with mean equal to expected observation \hat{z} of each particle - eq 3.14. With growing distance between expected and measured observation the particle weight decrease. The Q is a measurement covariance matrix saying how precise the measurement is. The k is index of particle in set of particles ψ .

$$w_t^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1}(z_t - \hat{z}^{[k]})\right) \quad (3.14)$$

The measurement covariance Q takes into account previous covariance matrix of EKF representing position of given landmark and measurement noise as described by equation 3.15.

$$Q = H \Sigma_{j,t-1}^{[k]} H^T + Q_t \quad (3.15)$$

3.3.3 Update EKF for observations

Updating the EKF model for observation is a bit more complicated as we need to deal with situation when the landmark was observed for the first time. In such a case we have to initialize EKF for given landmark. This step is usually tightly connected with prior step of weight update. Getting observation j in time t we get following initialization procedure for set of N particles that are indexed by k :

$$\begin{aligned} \mu_{j,t}^{[k]} &= h_{inv}(x_t^{[k]}, z_t) \\ H &= h'(\mu_{j,t}^{[k]}, x_t^{[k]}) \\ \Sigma_{j,t}^{[k]} &= H^{-1} Q_t (H^{-1})^T \end{aligned} \quad (3.16)$$

The $\mu_{j,t}^{[k]}$ and $\Sigma_{j,t}^{[k]}$ are parameters of EKF. The H is jacobian observation model h ([34]) at point $\mu_{j,t}^{[k]}, x_t^{[k]}$. Note that we assume that we process only one observation in one iteration of the algorithm. If we need to process more observations, we will repeat the iteration with same pose of the robot. If the landmark was observed for the first time, weight of the particle will be initialized by default value $w^{[k]} = p_0$ as we don't have mean of the EKF with which we could compare the observation.

If the landmark was already observed in the past we will update EKF according to following set of equations:

$$\begin{aligned} \hat{z}^{[k]} &= h(\mu_{j,t-1}^{[k]}, x_t^{[k]}) \\ H &= h'(\mu_{j,t-1}^{[k]}, x_t^{[k]}) \\ Q &= H \Sigma_{j,t-1}^{[k]} H^T + Q_t \\ K &= \Sigma_{j,t-1}^{[k]} H^T Q^{-1} \\ \mu_{j,t} &= \mu_{j,t-1} + K(z_t - \hat{z}^{[k]}) \\ \Sigma_{j,t} &= (I - KH) \Sigma_{j,t-1}^{[k]} \end{aligned} \quad (3.17)$$

For all features that are not observed the EKF simply is not updated.

3.3.4 Resampling - generating new set of particles

For resampling there can be used several algorithms. Probably the most straightforward is bin algorithm where each particle has its „bin“ - an interval in range $< 0, weight_{sum} >$. The size of the partical is naturally given by weight of the particle. All the intervals are places one next to another to form integral space without gaps. Than values from interval $< 0, weight_{sum} >$ are generated N-times, where N is desired number of particles in new generation. Particle is chosen into the new set if random value hits its interval. Particles with high weight can be selected several times and particles with low weight will be selected only one time or completely skipped.

3.3.5 Features of FAST SLAM algorithm

The FAST SLAM is effective in low-dimensional space that can be effectively covered by particles (typical for particle filter based algorithms). Its computational demands doesn't grow with growing set of mapped landmarks - it is only affected by number of particles. At this point independence of models for particular landmarks proves itself as very important feature. Probably the most important feature coming from particle based nature of the algorithm is effective dealing with data association. FAST SLAM implements by design multi-modal belief. Each particle has its own association of observations to landmarks. If the association is incorrect the particle will sooner or later naturally die due to low weight.

Disadvantage of FAST SLAM is that robot pose has no uncertainty. The uncertainty is modelled by set of particles byt one particle with highest weight represents the robot pose and this particle represent particular coordinates with no uncertainty. This disadvantage in combination with noisy motion model leads to generating a lot of particles that are later disposed due to non-matching onbservations. This problem is significantly reduces in FAST SLAM 2.0 ([23]) algorithm which takes into account also observations when predicting new particle poses as described by equation 3.18.

$$x_t^{[k]} \sim p(x_t | u_t, x_{t-1}^{[k]}, z_t) \quad (3.18)$$

Chapter 4

Actual approaches to outdoor localization

Large areas with small amount of usable markers are a special case for outdoor localization and SLAM. In general most of the methods count with at least one observable landmark any moment in time. This often does not apply to large sparse areas.

4.1 Categories of actual approaches

Actively broadcasting external localization systems: Most of actual approaches rely on external system that provide information for localization. These external systems usually compounds of network of nodes that act as some kind of beacons. Mobile robot measures either distance (lateration) or angle (angulation) to these nodes. After obtaining enough informations the robot solves a set of equations to find solution - its location. This category covers GNSS, wireless network localization and beacon based localization systems like VOR.

Marker based localization: Another category of localization approaches use observation of external markers - natural or artificial. Natural markers are markers that are recognized in the environment by the mobile robot itself. The markers are natural part of the environment. Their recognition and association is up algorithms used on mobile robot. On the other hand the artificial markers can be recognized easily - they usually come together with sensoric system and algorithm for their recognition and identification. More details about artificial marker based localization can be found in [4.6](#).

Localization based on external observation: Localization based on external observation uses external cameras or other sensors capable of observing the mobile robot and measure its position in defined area. More about this category of systems can be found in [4.5](#).

Other approaches: This category cover the approaches to outdoor localization that are not covered by prior categories. They are usually specific for particular tasks. One example belonging into this category is wired navigation for autonomous lawn movers. This approach is based on sensing of electromagnetic field by coil. The coil works as a sensor. The wire circuit placed under ground denotes the area in which

the mover can operate. The mover does not localize itself in this area but it can detect crossing the border line so it can always stay inside. Similar approach is used for navigating transport machines in industrial facilities. The wire in the ground or under ceiling serves as a guidance for mobile machines. The guiding wire may serve also as a communication channel between the mobile machine and operation center.

4.2 GNSS

First in category of localization solutions that rely on external system is satellite localization. Today there are several existing satellite networks including GPS (USA) [7], Galileo (Europe) [7], GLONASS (Russian federation) [7] and BeiDou (China) [2]. All these systems use trilateration to estimate position on the surface of the Earth. Details of GPS principle were described in chapter 2.2. Generic description of the GPS applies more or less also to other systems.

4.3 Localization using wireless networks

Using network of wireless nodes is another approach to localization ([28]). The nodes can communicate with each other using wireless technology based usually on radio signal. By analyzing properties of the signal the nodes can estimate location of rover traveling amongst them. Location using wireless networks is relative location - we can find relative location to nodes of the network. With additional information about node positions in absolute coordinate system we can compute absolute position of the rover (mobile device with receiver).

From principal point of view there are three approaches to finding relative location of the rover to wireless network nodes as described in [26]: trilateration, angulation and combination of both. The trilateration measures distances to nodes of wireless network. Knowing three or more distances unique position can be computed. Trilateration is visualized by figure 4.1. A three nodes A,B,C with known location are placed in area. The rover R travel through the area and measures distances from them. Getting three or more distance measurement the rover can compute its position as intersection of circles with diameters equal to measured distances. Having three measured diameters r_A, r_B, r_C and knowing positions of the nodes A,B,C the position can be computed by solving set of equations 4.1. The set of three quadratic equations with two unknown variables is redundant. Solving two equations together will give us two roots - each of them lies on intersection of circles with given diameters. This situation is visualized on figure 4.1 by possible rover positions R and R' . The third equation is used to eliminate invalid root so we get unique solution.

$$\begin{aligned} (R_x - A_x)^2 + (R_y - A_y)^2 &= r_A^2 \\ (R_x - B_x)^2 + (R_y - B_y)^2 &= r_B^2 \\ (R_x - C_x)^2 + (R_y - C_y)^2 &= r_C^2 \end{aligned} \tag{4.1}$$

The triangulation measures angles between wireless nodes and rover. Problem of localizing rover by measured angles is also called resection problem. Knowing three or more angles to nodes with known positions we can compute exact position of the rover. The situation is depicted in figure 4.2.

There are several approaches of computing position of the rover knowing angles to nodes. The [31] compares 18 resection algorithms. One of effective resection algorithms

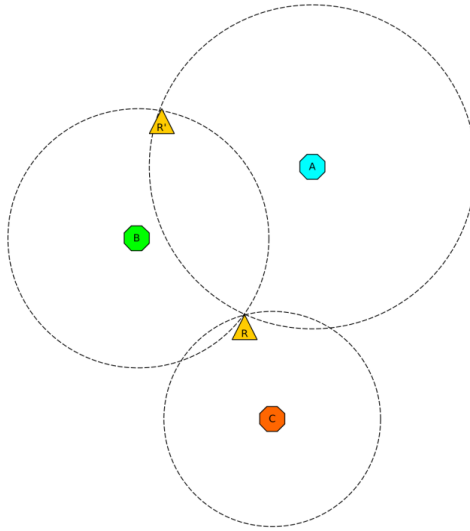


Figure 4.1: Trilateration principle.

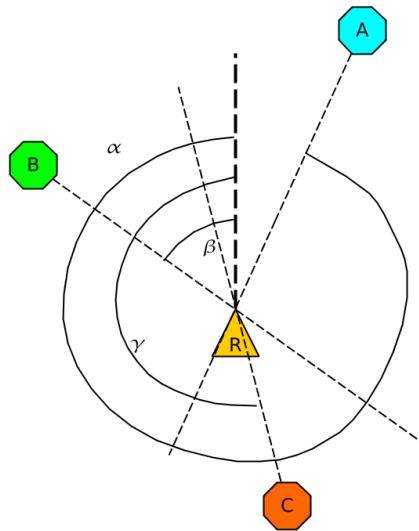


Figure 4.2: Triangulation principle.

called ToTal is shown in algorithm 1. The algorithm is described in the detail in [32].

Data: $\mathbf{A}, \mathbf{B}, \mathbf{C}, \alpha, \beta, \gamma$

Result: \mathbf{R}

Computation of delta coordinates:

$$\mathbf{A}' = \mathbf{A} - \mathbf{B}$$

$$\mathbf{C}' = \mathbf{C} - \mathbf{B}$$

Computation of cotangens:

$$\cot_{AB} = \cot(\beta - \alpha)$$

$$\cot_{BC} = \cot(\gamma - \beta)$$

$$\cot_{CA} = \frac{1 - \cot_{AB}\cot_{BC}}{\cot_{AB} + \cot_{BC}}$$

Computation of modified circle centers coordinates

$$x'_{AB} = A'_x + \cot_{AB}A'_y; y'_{AB} = A'_y - \cot_{AB}A'_x$$

$$x'_{BC} = C'_x + \cot_{BC}C'_y; y'_{BC} = C'_y - \cot_{BC}C'_x$$

$$x'_{CA} = (C'_x + A'_x) + \cot_{CA}(C'_y - A'_y); y'_{CA} = (C'_y + A'_y) - \cot_{CA}(C'_x - A'_x)$$

$$k = A_x C_x + A_y C_y + \cot_{CA}(A_x C_y - C_x A_y)$$

$$D = (x'_{AB} - x'_{BC})(y'_{BC} - y'_{CA}) - (y'_{AB} - y'_{BC})(x'_{BC} - x'_{CA})$$

Position of the rover

$$R_x = B_x + \frac{k(y'_{AB} - y'_{BC})}{D}$$

$$R_y = B_y + \frac{k(x'_{BC} - x'_{AB})}{D}$$

return(\mathbf{R})

Algorithm 1: ToTal triangulation algorithm

Most of localization systems use triangulation or trilateration technique to compute position of the rover. In some cases it is necessary to find only topology of the wireless network. Wireless signal analysis is also helpful in reconstruction of topology and locations of wireless nodes. For topology discovery algorithms based on challenge and response are used such as TopDisc [11]. To achieve a more precise localization of nodes a more sophisticated signal analysis is typically used.

Localization algorithms in wireless networks use signal strength (RSSI) [18] or time-based distance measurement using measuring of round-trip time, phase shift of pseudorandom code or phase shift of carrier. Localization based on RSSI use principle of trilateration to localize the rover. The RSSI ranging use relation between transmitted power of the transmitter and received signal power. As radio wave spread with constant energy and spherical surface of wavefloor grows quadratically the intensity of the signal in every single point of the wavefloor decrease. Relation between intensity and distance from the source of waves is given by relation 4.2. From this relation we can express the distance 4.3.

$$I = \frac{P}{4\pi r^2} \quad (4.2)$$

$$r = \sqrt{\frac{P}{4\pi I}} \quad (4.3)$$

In real world with obstacles the signal is attenuated by passing through the obstacles and spreads through different paths due to reflection. There is also noise from other wireless

traffic that interferes with the signal. This is the reason why naive approach has usually poor results. There are several models of the environment that model realistic spreading of the signal. With environment models the precision of RSSI localization can be improved to sub-meter precision under ideal circumstances. Another problem is that the decrease of the amplitude due to distance is non-linear. In larger distances the resolution of RSSI in dB/m decrease.

Distance measurement based on time difference is in general more robust and more precise. One of the most straightforward time measuring principles is measuring the round-trip time. If we have a network of nodes with well known location we can measure distances to particular nodes and compute the distance using trilateration as mentioned in [39]. Localization in cellular network is convenient because there are existing networks for mobile telephony and data transfers. In today's crowded world even the round-trip time itself is sometimes insufficient. The [20] suggests augmentation to the RTT-based approach using Bayesian inference method combining SINR (Signal to Interference plus Noise Ratio) measurements with RTT or machine learning approach when there is a map of signal strength obtained by supervised learning method.

The RTT or time delays in general can be measured using several approaches. One of them is phase shift of sent and received code sequence. The sender sends usually pseudo-random code sequence, receiver receives the sequence and sends it back immediately. The sender receives the returned code sequence and iteratively computes correlation of sent and received sequence. With each iteration receiver shifts the received sequence one bit forward and computes correlation with sent signal. When the correlation is highest the time delay is computed as number of shifts multiplied by code symbol length. The round-trip time is a sum of signal travel to receiver, receiver processing time and signal travel from receiver back to sender. Receiver processing time may vary. This source of imprecision can be eliminated by measuring the RTT several times and averaging it. Still the precision of code phase shift is limited by length of code symbol. To achieve better resolution some solutions use phase shift of carrier signal. As a code symbol is represented by two or more periods of carrier signal using carrier signal brings much better resolution. On the other hand carrier signal processing brings harder demands on signal processing [38].

4.4 Localization using beacons

Principal of localization using beacons is very similar to localization using wireless networks. The difference is that the beacons are designed for purpose of localization - in contrast to GSM cellular network localization where the network is intended for a different purpose and it is just „abused“ for localization. Example of such solution is VHF omnidirectional range (VOR) [6]. VOR is a beacon based localization system that uses angulation to estimate position of mobile rover. It was designed for aerial navigation and it is still a primary purpose of VOR. To measure angle from VOR beacon to rover the beacon transmits a two signals - one is omnidirectional and the another is a directional signal. The rover detects the highest amplitude of the directional signal and in this moment it measures phase shift of the directional and omnidirectional signal. With several angular measurements the actual location of the rover can be computed.

4.5 External visual localization

Another category of localization systems are systems based on observing mobile rover by cameras [15] or other optical sensors like laser rangefinders [33]. Systems based on cameras use two or more cameras for measuring position of the rover. The rover is detected in camera image and angle to the rover is computed. Knowing exact positions of the cameras and angle between camera centers and the rover the rover's pose can be computed using angulation principle. Of course the localization system needs to have some prior knowledge of the rover's shape and texture to be able to distinguish it from the background. The rover is often equipped with special markers to ease the detection. With sufficient resolution of localization system and sufficient amount of distinguishable landmarks it is possible to track motion of particular parts of the rover. Systems like VICON [1] allow detailed motion tracking of particular parts of the rover. For precise motion tracking and localization the image from cameras is often combined with depth information provided by depth scanners or 3D LIDARs.

4.6 Marker based localization

This method of localization uses prepared markers installed in the environment. The markers are usually in the form that can be easily recognized and distinguished from each other [22]. The localization system is installed on the rover. The rover detects markers in the environment and recognizes particular markers. According to relative position of the rover to the marker and prior knowledge of position of the marker the position of the rover can be computed. If the markers are supposed to be observed by camera they have predefined shape and color and they contain some kind of code that allows distinguishing particular markers. Example of such marker is QR-code or barcode.

Another method of localization based on markers is laser reflection guidance [4]. This method uses reflection of laser beam from markers adapted to particular wavelength of the laser. This way the markers can be easily distinguished from background. This approach may use angulation and lateration principle together to improve precision and robustness of the system. Despite that this solution can be used in outdoor environments typical application of this solution is indoor industrial environment like warehouses, docks and large factories. Poses of markers are known to the rover so it can easily estimate its location when a marker is detected. Reflective markers are usually installed under the ceiling of the hall so they can be observed from most places of the area. Thanks to good observability the amount of markers can be significantly lower without affecting robustness of entire solution.

RFID [12] is a technology of wireless exchange of informations between initiator of communication - interrogator in terminology of RFID - and communication slaves - the transponders. The transponders can be very small devices. There are three variants of RFID transponders: active, battery assisted and passive. The active transponders have their own power source and work as a common wireless communication devices. They listen to the wireless communication and when they are challenged they respond with requested information. All parts of the active transponder are powered from its own power source.

The battery assisted transponders have still its own source of power but this source is usually very minimalistic with limited power. The power source only powers the internal control circuit of the transponder but not the RF part. Energy for wireless communication is taken from interrogator. When interrogator challenges a transponder it broadcasts the

request in form of electromagnetic wave. This wave is modulated and carries the content of the challenge but it also transports energy and thanks to this it can be also considered as a minimalistic power source. Energy of the wireless signal is sufficient for the transponder to power its own transmitter and transmit the information.

The last variant of transponder is passive transponder. It has no own source of energy. When the challenge from the rover is received the power of the signal is used to power transmitting part and also the control logic of the RFID transponder. This small amount of energy is sufficient to interpret the challenge from the interrogator, generate response and transmit the response back to the interrogator. Of course in case of battery assisted and passive RFID transponders the reach of the response is very small - tens of millimeters usually. Advantage of passive RFID transponders is their size. The antenna needs to be large enough to receive enough of energy but it can be completely flat. With small internal chip the RFID transponder can be in form of a tag or label.

For purposes of localization the RFID transponders are installed at particular positions in the environment [9]. They can be observed using the RFID interrogator. Short range of RFID communication is advantage because it allows the rover to estimate its position according to observed RFID tag. With greater range the position estimation would be less precise. Disadvantage of short communication range of RFID tags is that rover discovers them only when it crosses over them. This is why are the RFID tags used for localization usually installed on the ground.

There are also less sophisticated but very precise methods of localization that can be included to marker based localization. One of them is guidance line observed by optical reflection sensor. This approach is used usually in indoor environments with flat floor where the line can be easily detected. Equivalent of this approach in outdoor environment is wire guidance using induction loop. The line is replaced by wire placed few centimeters under ground. The wire makes a loop that is powered by pulse power supply. Instead of using reflection sensor the wire is detected by system of coils. These approaches are described in [29].

4.7 Limitations of actual methods

It is not a surprise that each approach to the SLAM have its limitations. Particular methods based on algorithms described above fulfill particular requirements while they fail to fulfill others. Developing an universal SLAM method that would work in arbitrary environment is still an ultimate goal. Some of the methods work only in a flat terrain as they use 2D mapping, others based on 3D laser scanning with voxel grid map are very robust in almost any environment but very resource demanding so they can not be easily used for very large environments. Moreover 2D or 3D laser scans do not work very well in large flat areas with a very few objects with distinguishable geometry. When there is no observable object in reach of sensors the SLAM algorithm can not update posterior and has to rely on dead-reckoning only which is usually not very reliable - especially in hard terrain.

Another approach is to use localization based on network of external navigation nodes. This include global navigation satellite systems with network of satellites like GPS and network of wireless nodes that help with the navigation. This category of localization methods have usually the best performance. Position error is limited by network properties so systematic error can be avoided. Also solving problem of lost robot is trivial with localization in network active nodes. Localization with wireless network could be also energy efficient if the robot only passively listens to signal broadcasted by nodes of the

network. This all applies to the situations when robot can receive signal from the network. Once the signal is cut off the robot has to rely on dead-reckoning from last known position. Despite these limitation the localization using some kind of wireless network is the most frequently used in today's world.

There is also another problem limitation of wireless network localization. This mode of localization usually uses N-lateration to localize the mobile rover. Despite the trilateration provides very good estimation of pose in the coordinate system of the network it principally can not provide estimation of orientation. Of course if the mobile rover moves we can assume that it is moving ahead so we can estimate where is it heading but this solution can not recognize if the mobile rover is moving backward or strafe left or right. This problem can be partially mitigated with fusing wireless network or satellite based localization with inertial measurement unit and odometry as described in [27] but in extreme case when the rover would spin on a spot the error of orientation estimation would grow without limits. There are existing systems that use N-angulation instead of N-lateration - particularly VHF Omni Range (VOR) [6]. Despite ongoing upgrades the network of VOR beacons does not offer precision comparable with GPS and other global navigation satellite systems.

Passive artificial landmarks do not differ from localization methods that use natural landmarks but they differ in overall properties. Artificial landmarks are usually a way more reliable than natural ones - this is why they are being used. They can be reliably detected and usually also reliably associated so it helps to mitigate the re-observability and association problems. If the pose of artificial landmarks is known in advance they can also limit the maximal error of mapping - even warped map can be corrected by observing landmark with known pose. The limitation of this attitude is that network of artificial landmarks has to be created before the robot can localize. And in outdoor environments it has to be maintained. It brings additional effort if the landmarks are passive and can not report its state as satellites or wireless network nodes.

Another option is external system that observes the mobile robot and estimates its location according to the observations. The localization is not a matter of the robot but it is a matter of a network of nodes that observe particular area in which the robot operates. The robot can be equipped with special beacons that are detectable by nodes in the observation network to improve reliability. System like this can offer a very good precision of localization. With properly configured network it can reach a millimeter precision. On the other hand establishing and maintaining the network is usually rather time consuming and expensive task. Practically the observation networks are usually rather small. The network is the most significant disadvantage of such system. Example of such system is VICON [1]. Advantages are good precision, independence on robot sensoric subsystem so operation of the robot does not affect localization precision. Disadvantage is dependency on observation network and also maintenance of this network.

Chapter 5

Goals of this work

According to limitations of existing methods described above goals of this work were declared. Instead of trying to create an ultimate solution for SLAM that would work in arbitrary environment which is a goal that is really hard to achieve I decided to cover a gap in variety of existing solutions. Most of the outdoor localization solutions aim on localization in relatively dense environments with artificial structures like houses or roads. This finding helped me to define the primary goal of this work.

Develop solution of localization in large outdoor areas: The primary goal is to develop a SLAM solution that would work in large outdoor environments with only few objects that can be considered as landmarks.

Independence on external systems: Another very important goal is to create a self-contained SLAM solution. By self-contained I mean a solution without any external supportive systems (active or passive) - all the sensors used for SLAM are installed on the robot itself. Despite there are satellite navigation systems with good precision it is sometimes not possible to receive GNSS signal.

Improve robustness of dead-reckoning: As my solution is supposed to operate in areas with only few distinguishable objects the moments when no landmark is in sight of sensors will not be rare. Third goal is to improve dead-reckoning to make it more reliable in such situations.

5.1 Proposed solution for large outdoor SLAM - Basic principles of the approach

As the SLAM solution is supposed to work in large areas it is necessary to maintain a map of the area. There are no assumptions about terrain so it needs to maintain map of the environment in 3D. Feature-based SLAM method best suits these requirements. FAST-SLAM algorithm was choosed as the core of SLAM algorithm for this solution. Map is represented by set of guassians that allow to model uncertainty. Pose hypothesis is represented by particle and uncertainty of pose is represented by set of particles.

Very important is question of markers that should be detected by this solution. The environment is an area of large size with only few objects in it. Typical aspect of he objects is that they protrude above the terrain. Finding these objects by 3D laser scan is difficult as they are too far for most of sensors with reach in tens of meters and what is more important



Figure 5.1: Sensoric system installed on experimental robot RUDA.

even if the sensors could reach the objects the density of distance measurements would be probably too low to find many of objects we are interested in. Typical object that protrude above ground in large natural environment is a tree. As the branches with leaves are not very reliable marker because of reflections and partial opacity the most interesting part of the tree is the trunk. As the trunk has a cylindrical shape it can be observed from other directions without significant loss of precision. So amongst the artificial objects it would be useful and in some cases necessary to find tree trunk and measure distance to it.

This requirement limit selection of sensors used for the task. As we can barely afford to cover environment with distance measurements due to reach of sensors and density of scans the only way is to measure distances selectively. To make selective measurement we need to find the object first and then point the rangefinder at it. Finding objects in 3D environment is not easy. Scanning the environment blindly with a rangefinder would be too slow. To find the objects a visual-based method was chosen. The object can be found in camera view and then the long range single point rangefinder can be pointed to it. Only laser rangefinders offer required reach with necessary precision to measure distance to one particular distant object. To orient the rangefinder it has to be installed on a pan-tilt manipulator.

To overcome imprecision of the manipulator the camera has to be installed on it too and aligned with the rangefinder. This was the sensoric system solution used for this work: Camera with single point laser rangefinder aligned together (parallel axis) installed on top of P-T manipulator. Photo of sensoric system installed on robot RUDA that was designed and constructed at Faculty of Information Technology at Brno University of Technology can be observed in image 5.1.

One of the goals is improvement of precision and stability of dead-reckoning. Dead-reckoning is also important part of entire solution. Basic input for dead reckoning is odometry based on geometrical model of robot chassis. This can be used as initial estimate but to achieve a good precision odometry has to be fused with additional sensors. Odometry is

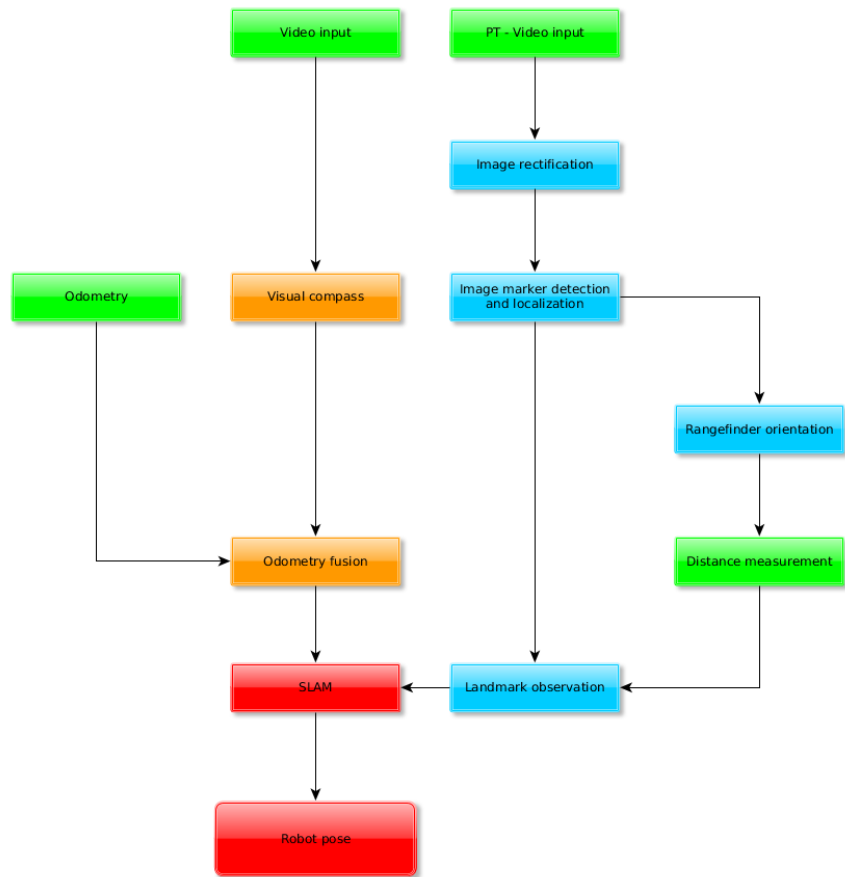


Figure 5.2: Data flow in sensoric system.

relatively precise for linear motion but rotational precision has to be improved. To achieve this visual was chosen as it is not a pure dead-reckoning but it takes into account surrounding of the robot and thus it has potential of better robustness. Many visual compass solutions are based on detecting particular features between consecutive frames to estimate rotation difference. In the large relatively empty environment it could be difficult to find reasonable features. This is why a feature-less approach was chosen.

In proposed solution the odometry fused with visual compass serves as input for motion model of SLAM algorithm. Uncertainty of the fused odometry is modelled by motion model to obtain more precise proposal distribution. Structure of entire solution is depicted in figure 5.2.

Chapter 6

Conclusions

This thesis deals with actual problems of robot localization in outdoor environments. Particularly it aims on large environments with only a few landmarks. Frequently used solution is based on satellite network or network of beacons. Unfortunately this approach can not be always used as it relies on external infrastructure that might become unavailable for some reason. The reason could be failure of the infrastructure or some kind of attack against it. Moreover there are other use cases when robots can not rely on external infrastructure. Typically exploration of other planets is an example of such cases. This is why localization without external systems was chosen as a goal of this work.

Generic requirements for the solution were defined in chapter 5. Deeper analysis of the requirements helped to design the solution proposed in this thesis. Essential problem is that commonly used attitudes of observing the environment do not provide usable data. There are problems with range of the ranging sensors, density of range scans and also with quantity of sensoric data. To overcome this problem new solution of sensoric system was designed that is based on camera and laser rangefinder. Camera is used to analyze the environment and find potentially useful objects and laser rangefinder is used to obtain distance measurements.

Requirements to maintain large maps with sparse landmarks shows that it is convenient to use feature based map representation. In feature based maps every landmark can be represented as object with location, orientation, representation of uncertainty and possibly some other attributes. Compared to grid cell based maps this representation allows saving resources needed to represent the map and allows more advanced representation.

The proposed solution compounds of three independent parts: Visual compass, Landmark detector and SLAM algorithm implementation. The visual compass is supposed to provide more precise values of robot orientation. This orientation data together with odometry should improve dead-reckoning so the robot should be able to travel hundreds of meters without observation of any landmark and not getting lost despite it.

Input for the localization relies on data provided by landmark detector. Quality and quantity of the data is essential for reliable localization. The localization itself is based on Fast-SLAM algorithm with feature based map. This algorithm was tested in many application and proved its robustness. The solution is modular - the landmark detector is a module that can be replaced or extended with another module running simultaneously to allow localization in different environment.

Implementing the solution described in this thesis should allow a robot in outdoor environments with only few landmarks to localize without support of external systems.

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