

Doctoral thesis (hereinafter referred to as "thesis"), title of the thesis:

High order numerical method in modelling and control systems

Name of the doctoral student (hereinafter referred to as "student"), name and surname:

Ing. Petr Veigend

Name and institution of the reviewer (full name of the reviewer, full name and country of the institution):

**Doc. Ing. David Horák, PhD., associate professor
VŠB-Technical University of Ostrava & Institute of Geonics**

Please state your opinion on the following aspects of (I) the student's thesis and (II) the student's overall achievements, and (III) state your conclusion (a minimum of approx. 300 characters for each item below is recommended):

I. Thesis

Appropriateness and relevance

Is the area addressed by the thesis appropriate to the particular scientific discipline of the thesis and does the thesis address relevant problems within the chosen area?

The thesis deals with an essential topic of numerical solution of initial-value problems of ordinary differential equations. Valuable results are then related to the high order numerical methods in modelling and control systems. Although they suffer from large number of operations disabling their usage in real-time problems, the results in the thesis demonstrate, that they are able to beat standard state-of-the-art methods.

A summary of the contributions of the thesis

From your point of view, please summarize what the goal of the thesis is, what the main contributions of the thesis are, and whether the thesis has achieved the chosen goal.

Please indicate also specific contributions of the student.

The main contribution is the comparison of numerical solution techniques for ordinary differential equations, analysis of properties of the higher-order variable-step variable-order numerical method based on the Taylor series - so called Modern Taylor Series Method (MTSM), its extension for solving nonlinear problems, its further optimizations and demonstration that its usage can be the most efficient considering speed and accuracy of computations of various problems, especially then of control system problems. If among chosen goals belong those presented in Section 1.1 and Chapter 7, then the reviewer pronounces, that they were fulfilled.

Novelty and significance:

Please assess the level of novelty of the results and their significance for the given scientific area, for its further development, and if applicable for possible applications in practice.

The novelty consists in application of the MTSM method in real-world problems (Chapters 3 and 4) and its optimization especially for problems from the area of control and regulation (Chapters 5 and 6) for both, linear and nonlinear types. In case of nonlinear problems, the chain rule is applied for multiple-function multiplications so that matrix-vector representation is obtained and reusing pre-calculated terms and step-size control is applied to reduce operations number. The performance of the MTSM method is discussed and illustrated by graphs. Reviewer appreciates presented comparisons with state-of-the-art ODE solvers but misses the detailed theoretical and experimental performance and error analysis, e.g. number of steps and operations and computation times depending on step-size h , parameter h_scale and given tolerance. Absolute stability area increases proportionally to the number of terms of the Taylor series which could be used in variable-precision arithmetic. Figures 4.37 and 4.39 illustrate nicely how MTSM automatically uses larger order (more terms of the Taylor series) in rapidly changing parts of solutions.

Question: What is the difference between TOL, MTSM_TOL and ERROR (see e.g. Tables 3.4, 3.7)?

Evaluation of the formal aspects of the thesis:

Please evaluate formal qualities of thesis and its language level.

The thesis is written in well understandable English, its quality is very high, there is minimum of typos. Visual form is also on high level including mathematical expressions, presented tables and graphs. Thesis could be improved as it is illustrated by following examples:

Page 18:

In the first equation there appears f''

It is missing, what D_x denotes in the second equation

Page 19:

In eq. (2.5) spaces are missing

In eq. (2.6) y should be bold

In Def 1 f is written using standard form, in Theorem 1 using bold

Page 20:

two different ways of writing an interval $[a,b]$ vs $\langle a,b \rangle$

Page 21:

In Theorem2 and text further, f , P_N , R_N are functions of t , not of x

Page 29:

There is a mismatch in using N and n

Review of a Doctoral Thesis at FIT BUT

Page 30:

How eq. (2.20) is related to the Laplace image of eq. (2.19)?

Page 38:

In eq. (3.4): $A^{(n-1)} \rightarrow A^{(N-1)}$

In thesis there is mismatch in denoting n -th derivative of the function f : $f^{(n)}$ vs $f^{[n]}$

Page 39:

In Pascal triangle n should be italic

Page 47:

Figure 3.4: it is not clear what is optimization1 and optimization2

Page 49:

In eq. (3.25): $g(t) \rightarrow g(b)$

Page 55:

In Table 3.3 and others it is not clear, what are tolerances for compared solvers and why MTSM_TOL differs by factor 10^{-3} from TOL

I miss the systematic experimental evidence of numbers of steps and operations, calculation times and errors for various reasonable h parameter values, similarly to Table 3.13. An effect of h_{scale} could also be demonstrated.

Page 75:

At eq. (3.58) a verb is missing – e.g. “is considered”

Page 77:

What does it mean: “time of calculation was very large”? – no times are presented in tables

Page 79:

In Tables 4.2, 4.3, 4.4, 4.5, 4.6, 4.8, 4.9, 4.10, 4.12, 4.13, 4.14, 4.15 ... the Ratios are related to time achieved using MTSM_orig or MTSM_opt? Then one ratio is missing, as runtimes of these methods are not equal. How the value of h_{scale} was chosen? Why the error for these two methods is not reported?

Page 80:

Problem B4 interesting $\rightarrow$ Problem B4 is interesting

Page 107:

In eq. (4.35) k is not from interval of real numbers but interval of integers.

Page 146:

$\omega r \in (1, 10, 100) \rightarrow \omega r \in \{1, 10, 100\}$

In Tables 6.6, 6.8, 6.9 etc. numbers of steps and set tolerances are missing

Page 175:

$x'_3 \rightarrow y'_3$

Page 177:

Why variants (6.56) and (6.57) are compared using various h and h_{scale} parameters?

Introduction of computational resources is missing – are they the same as those presented in Chapter 6?

Quality of publications

Has the core of the thesis been published at an appropriate level? Please judge the quantity and quality of the publications. When judging the quality, please take into account internationally recognized standards (WoS/Scopus quartiles, CORE ranks, specific knowledge of flagship publication channels of agiven community, etc.) in a way appropriate for the given area of the thesis.

Reviewer evaluates positively both, high quantity of 27 papers at delivered list reporting period 2015-2022 mostly related to the PhD topic, which is in accordance with the Scopus database, and their quality - 3 of them are published in journals with impact factor (1x Q2 in 2021 and 2x Q3 in 2018, 2019). As mentioned in section dealing with future work, reviewer expects ongoing publication activity. Publication activity is for the purpose of successful defence highly sufficient.

II. Student's overall achievements

Overall R&D activities evaluation:

Does the student's thesis, the results included into it, and possible other scientific achievements listed in the list of scientific activities indicate that he/she is a person with scientific erudition and creative abilities?

Student has proven by means of his thesis, achieved results, adapting and optimizing MTSM method in the area of control systems, numerical experiments, enclosed papers and associated activities his high research, creative, publication, programming and educational skills and abilities required for PhD degree.

Assessment of other characteristics (optional):

More characteristics of the student may be added here (e.g., awards, grant participation, international collaboration, etc.).

Based on student's CV also other his activities are mentionable - his cooperation with Vienna University of Technology within Action Czech-Austria project and his memberships in FIT organizations.

III. Conclusion

The conclusion should contain an explicit statement saying whether, in your opinion, the thesis and the student's achievements until now meet the generally accepted requirements for the award of an academic degree (in accordance with Section 47 of Act No. 111/1998 Coll., on higher education institution).*

* Short overview of both the Act and corresponding internal BUT regulations is enclosed.

Based on the reviewed PhD thesis and taking into account all student's publications and activities, I declare that student meets in accordance with Section 47 of Act No. 111/1998 Coll., on higher education institution all requirements for the award of an academic degree PhD and therefore I recommend the thesis for its defense.

Brno 03.11.2023

Signature of the reviewer: