

**Doctoral thesis** (hereinafter referred to as "thesis"), title of the thesis:

**High order numerical method in modelling and control systems**

**Name of the doctoral student** (hereinafter referred to as "student"), name and surname:

Ing. Petr VEIGEND

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**Name and institution of the reviewer** (full name of the reviewer, full name and country of the institution):

Priv.-Doz. Dr. Alexander SCHIRRER, senior scientist

Technische Universität Wien (TU Wien), Vienna, Austria

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Please state your opinion on the following aspects of (I) the student's thesis and (II) the student's overall achievements, and (III) state your conclusion (a minimum of approx. 300 characters for each item below is recommended):

**I. Thesis**

**Appropriateness and relevance**

Is the area addressed by the thesis appropriate to the particular scientific discipline of the thesis and does the thesis address relevant problems within the chosen area?

The thesis addresses topics which form the fundamental backbone of simulation methods – the numeric solution of initial-value problems of ordinary differential equations, so the addressed topics are highly relevant for many areas of science. Thus, the chosen research topic is also appropriate in the modelling and simulation domain, serves as a contribution to some extent in control theory and applications, as well as in scientific computing in general.

**A summary of the contributions of the thesis**

From your point of view, please summarize what the goal of the thesis is, what the main contributions of the thesis are, and whether the thesis has achieved the chosen goal.

Please indicate also specific contributions of the student.

The goal of the thesis is to develop, apply and demonstrate extensions to the Modern Taylor Series Method (MTSM) for the time-domain simulation of linear and non-linear system dynamics, with a focus on the real-time control context. Several research questions detail this goal and are elaborated within the thesis respectively in the comprehensive portfolio of publications of Ing. Veigend.

The thesis is clearly organized into introduction and integration method fundamentals (Ch. 1 and 2), the MTSM with extensions in detail (Ch. 3), followed by a range of illustrative and benchmark problems and their evaluation (Ch. 4). Then, the control perspective is outlined (Ch. 5) and accompanied by again a range of closed-loop (controlled) test and benchmark problems and their solution via MTSM (Ch. 6).

In particular, the thesis builds up on the previously existing MTSM and adds several contributions.

The core or main contribution of Ing. Veigend's work lies in the proposed range of extensions to the MTSM (outlined in Ch. 2 and 3): an assessment and extension of transformation methods to prepare arbitrary nonlinear high-order ODE systems into a form suitable for the MTSM, applying automatic differentiation and transformation techniques, optimizing/minimizing the number of terms in the pre-processed ODE representation, and providing optimized representations for selected compound function terms (2- to 5-factor multiplicative terms). Also, extensions towards variable-order precision arithmetics and basic structures for efficient hardware implementation of the arising operations during an MTSM solution are proposed and discussed.

The second contribution is that of providing an extensive reference of example problems and benchmarks to illustrate the MTSM application itself, as well as its relative (superior) performance compared with well-known state-of-the-art ODE solvers.

Finally, a contribution also lies in the control perspective that is taken, and the set of illustrative examples therein. In these results, the automatic (variable) time stepping and its beneficial effect in terms of solver efficiency is seen clearly, as known from other variable-step integration methods.

The thesis satisfactorily achieves the set research goals. The student's own contributions are and addresses

**Novelty and significance:**

Please assess the level of novelty of the results and their significance for the given scientific area, for its further development, and if applicable for possible applications in practice.

It is unusual today to encounter approaches, like those presented in Ing. Veigend's thesis, that are able to address a seemingly long-matured core topic of science – the numeric solution of ordinary differential equations – and provide striking results and performance that seems to dominate well-developed classical methods by a wide margin. This is done, in part, in this thesis, and this also requires special care and attention in critically reviewing and acclaiming the scientific work and its comparability and in assessing real-world performance.

One such consideration concerns the basis of comparison between the MTSM and other ODE solvers' solutions. Many application examples are shown throughout the thesis, with convincing results in terms of total computation time vs. state-of-the-art ODE solvers. However, it should be pointed out clearly that this only indicates the MTSM performance in problems whose solution is only sought on a coarse time grid (or at the end point). From the reviewer's understanding, providing an (interpolated) solution at any fine time grid could be possible in principle from the MTSM solution representation, but would require an additional post-processing step. This would allow a more direct and practically more relevant comparison of the solver performances (with an expectedly smaller runtime advantage of MTSM over the other methods).

Also, several theoretical passages (on integration stability and on control theory aspects) show some inaccuracies. These, however, do not damage the core contributions of the thesis.

Given the high potential and importance of the MTSM as a versatile computation method, it is desirable to deepen the rigorous, theoretical understanding of the method. Particularly, a systematic truncation error analysis and a deeper analysis of the stability region associated with the variable-order nature of the

MTSM would be desirable. Finally, the realtime aspect is treated in a purely empirical analysis fashion. As follow-up research it seems necessary to investigate possibilities to compute a fixed, limited number of operations and guarantee a bounded approximation error. This is not yet present and would increase usability in hard realtime settings.

Summing up, the topic is highly relevant and interesting, as the computational solution of ODEs lie at the very heart of scientific computation. The thesis provides insight into and novel results in terms of many relevant extensions for the MTSM to unlock its use in linear and nonlinear dynamics system simulation and gives many concrete examples. Also, a control perspective is attempted to highlight selected MTSM applications in a control context. The thesis also shows the way for relevant future work to be done in the subject to further increase the applicability of the MTSM in various application domains.

#### Evaluation of the formal aspects of the thesis:

Please evaluate formal qualities of thesis and its language level.

The shown English language level is appropriate and high. The extensive thesis text is carefully written, proofread, and practically free of typos. Also, the work is carefully elaborated and typeset clearly, including understandable mathematical expressions and many illustrative figures, but at times falls into colloquial language use and uses vague terminology. Still, in total, the thesis is a scientific work of generally high technical quality and high readability.

#### Quality of publications

Has the core of the thesis been published at an appropriate level? Please judge the quantity and quality of the publications. When judging the quality, please take into account internationally recognized standards (WoS/Scopus quartiles, CORE ranks, specific knowledge of flagship publication channels of a given community, etc.) in a way appropriate for the given area of the thesis.

The student's publication record shows a large number of related conference publications (24, from 2015 to 2022) as well as 3 journal articles (2018 in a Q3 journal, 2019 in a Q3 journal, and 2021 in a Q2 journal). This portfolio is consistent with the thesis structure and depth: the thesis contains many application examples and a broad spectrum of contributions, but does not provide deep theoretical results or proofs. Given the remarkable results outlined in the thesis, the thesis topic and the research field of (modern) Taylor series methods certainly would benefit from a rigorous treatment to explore the method's strengths and weaknesses systematically or deliver quality guarantees.

Considering the posed research goal and the research questions, the portfolio and breadth of publications (and also their distribution into the majority being conference papers) is highly satisfactory. Its wide range of topics and applications shows the versatility of the method, but it also highlights the future need to rigorously characterize the MTSM's properties.

## II. Student's overall achievements

#### Overall R&D activities evaluation:

## *Review of a Doctoral Thesis at FIT BUT*

Does the student's thesis, the results included into it, and possible other scientific achievements listed in the list of scientific activities indicate that he/she is a person with scientific erudition and creative abilities?

The student has elaborated an extensive, well-readable doctoral thesis with a broad range of remarkable and relevant results. The large number of conference publications indicate that the student realizes a high level of scientific dissemination and engages in scientific outreach and networking. The task of putting the MTSM context into a control perspective could mostly be achieved by the student, which also adds an inter-disciplinary aspect to his research work. Combined with the student's journal publications, the scientific standing fulfills the typical international requirements for a doctoral degree. The listed research activities indicate that the student is capable of scientific high-quality work and show the structural skills and abilities demanded from scientists.

### Assessment of other characteristics (optional):

More characteristics of the student may be added here (e.g., awards, grant participation, international collaboration, etc.).

Besides creating said scientific output, Ing. Veigend has shown broad engagement in many extracurricular activities. He is consistently involved in faculty programs to organize and develop teaching structures, curricula and support students. Particularly, over the last 5 years, he has filled the position of study advisor at TU Brno. He also actively participated in international study exchange programs (Aktion Czech-Austria) with the partner research group at TU Wien. Being consistently active in these fields is a high-valued quality in any researcher, and this generates benefits for the community, the national education system, and for international partnerships alike.

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### III. Conclusion

The conclusion should contain an explicit statement saying whether, in your opinion, the thesis and the student's achievements until now meet the generally accepted requirements for the award of an academic degree (in accordance with Section 47 of Act No. 111/1998 Coll., on higher education institution).\*

\* Short overview of both the Act and corresponding internal BUT regulations is enclosed.

After a thorough and careful review of the thesis, the related publications, and the student's other achievements, I am convinced that the student meets the generally accepted requirements for the award of an academic doctoral degree (in accordance with Section 47 of Act No. 111/1998 Coll., on higher education institution). I therefore recommend to proceed the dissertation process and allow the student to defend his thesis.

Vienna, 29<sup>th</sup> Sep. 2023

Signature of the reviewer: