BRNO UNIVERSITY OF TECHNOLOGY
VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

FACULTY OF INFORMATION TECHNOLOGY
FAKULTA INFORMAČNÍCH TECHNOLOGIÍ

DEPARTMENT OF INTELLIGENT SYSTEMS
ÚSTAV INTELIGENTNÍCH SYSTÉMŮ

LIBRARY FOR FINITE AUTOMATA AND TRANSDUCERS

KNIHOVNA PRO KONEČNÉ AUTOMATY A PŘEVODNÍKY

BACHELOR'S THESIS
BAKALÁŘSKÁ PRÁCE

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Zadání bakalářské práce

Řešitel: **Bieliková Michaela**
Obor: Informační technologie
Téma: **Knihovna pro konečné automaty a převodníky**
Library for Finite Automata and Transducers
Kategorie: Algoritmy a datové struktury

Pokyny:

1. Nastudujte teorii konečných automatů a převodníků.
2. Nastudujte efektivní algoritmy pro práci s konečnými automaty a převodníky.
3. Navrhněte knihovnu pro konečné automaty a převodníky, která bude vhodná pro rychlé prototypovaní nových algoritmů.
4. Implementujte navrženou knihovnu.
5. V knihovně implementujte nastudované efektivní algoritmy pro práci s konečnými automaty a převodníky.
6. Knihovnu otestujte na benchmarku zadaném vedoucím práce.

Literatura:

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- Body 1 a 2 a část bodu 3.

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Datum zadání: 1. listopadu 2016
Datum odevzdání: 17. května 2017

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Abstract

Finite state automata are widely used in the field of computer science such as formal verification, system modelling, and natural language processing. However, the models representing the reality are complicated and can be defined upon big alphabets, or even infinite alphabets, and thus contain a lot of transitions. In these cases, using classical finite state automata is not very efficient. Symbolic automata are more concise by employing predicates as transition labels. Finite state transducers also have a wide range of application such as linguistics or formal verification. Symbolic transducers replace classic transition labels with two predicates, one for input symbols and one for output symbols. The goal of this thesis is to design a library for letter and symbolic automata and transducers which will be suitable for fast prototyping.

Abstrakt

Konečné automaty majú široké uplatnenie v informatike, okrem iných vo formálnej verifikácii, modelovaní systémov a spracovaní prirodzeného jazyka. Avšak modely skutočne reprezentujúce realitu bývajú veľmi komplikované a môžu byť definované nad veľkými, v niektorých prípadoch až nekonečnými, abecedami, a teda môžu obsahovať veľký počet prechodov. V týchto prípadoch nemusí byť je použitie algoritmov na prácu s konečnými automatmi efektívne. Symbolické automaty poskytujú stručnejší zápis tak, že namiesto symbolov v prechodoch používajú predikáty. Konečné prevodníky tiež majú široké uplatnenie, od ligvistiky až po formálnu verifikáciu. Symbolické prevodníky nahradzujú symboly dvojicou predikátov — jeden predikát pre vstupné symboly a jeden pre výstupné. Cieľom tejto práce je návrh knižnice pre klasické a symbolické automaty a prevodníky, ktorá bude vhodná na rýchle prototypovanie nových algoritmov.

Keywords

finite state automata, symbolic automata, transducers, efficient algorithms, formal verification

Klíčová slova

konečné automaty, symbolické automaty, transducery, efektívne algoritmy, formálna verifikácia

Reference

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Library for Finite Automata and Transducers

Declaration

Hereby I declare that this bachelor's thesis was prepared as an original author's work under the supervision of Ing. Martin Hruška. All the relevant information sources, which were used during preparation of this thesis, are properly cited and included in the list of references.

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Michaela Bieliková
May 15, 2017

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I would like to thank my supervisor Ing. Martin Hruška for his dedicated guidance, numerous advice and immense patience.

Contents

Chapter 1

Introduction

Finite automata are used in a wide range of applications in computer science, from regular expressions or formal specification of various languages and protocols to natural language processing [16]. Further applications are hardware design, formal verification or DNA processing. In formal verification, finite automata are used for a representation of system configurations. In formal specification, they are used to describe a complex process (such as communication between client and server) in a general, formal and concise way. This thesis further deals with finite transducers which are also widely used in computer science. Transducers have application in natural language processing where they can describe phonological rules or form translation dictionaries or in formal verification to model system behaviour [6, 14, 16].

In many practical applications, large alphabets are used which leads to big number of transitions and quick increase of computing demands. Furthermore, the common forms of automata and transducers cannot handle infinite alphabets. In practice, big alphabets are widely used in natural language processing, where an alphabet must contain all symbols of a natural language. Languages that derive from Latin alphabet such as English usually contain less than 30 symbols, but with the use of diacritic, this number can grow to double. This number further increases in alphabets that have a symbol for every syllable such as Chinese or Japanese. Even larger alphabets may appear in applications in which the symbols are words. Electronic dictionaries often have more than 200K words and even this number is not enough to handle unrestricted texts. Moreover, robust syntactic parsers often require an infinite alphabet [14]. In these cases, a modification of automata using predicates instead of elementary symbols can be used. Predicates represent a set of symbols with one expression and therefore can reduce the number of transitions. This formalism is known as symbolic automata and transducers. As it will be shown, the most of the operations used on finite automata are easily generalizable for symbolic automata and transducers. The main goal of this thesis is to design a library for symbolic automata and transducers which will be suitable for fast prototyping.

Moreover, the library will provide structure for classic letter automata and some of the state-of-the-art algorithms for language inclusion checking. Important operations over automata and transducers, such as minimisation and language inclusion (often used in formal verification) need the automata to be determinised first, which can exponentially increase the number of states or transitions. Fortunately, advanced algorithms that allow inclusion checking without determinization, exist. This thesis employs two heuristics for language inclusion checking algorithms based on simulations [9] and antichains [5, 10]. Simulations examine the language preserving relation of each pair of states and then eliminate the states

which have the same behaviour for every possible input symbol. This allows reducing the number of states and therefore enables more efficient manipulation with the reduced automaton. Antichains are mostly used in language inclusion and universality checking which can be decided by finding counterexamples. The main idea behind antichains is that if we have not found a counterexample in a small set of automaton states, it is not necessary to look for a counterexample in a superset of these states. Including more states in the checked set cannot reduce the accepted language. The superset of an already checked state widens the accepted language and therefore cannot contain a new counterexample. Formal definitions and algorithms for simulation and antichains as well as their usage in symbolic automata can be found in Chapter 4 of this thesis.

Currently, there are more libraries that deal with some form of symbolic automata. The first ones are AutomataDotNet in C# by Margus Veanes [17], and symbolicautomata in Java by Loris d'Antoni [7]. While these libraries are efficient and offer many advanced algorithms, such as determinization, minimization and simulations, they are very complex, have a slow learning curve, and therefore are not suitable for quick prototyping of new algorithms. Further, there is the VATA library[4] in C++, which is a high efficient open source library. VATA offers basic operations, such as union and intersection, and also more complex ones, such as reduction based on simulation [9] or language inclusion checking with antichains optimisation [5]. It can also handle tree automata. It is modular and therefore easily extendible, but since it is written in C++, prototyping in VATA is not easy. FAdo library[1] is a library for finite automata and regular expressions written in Python. Unfortunately, it is a prototype, immature and not well documented. Very fast and efficient library is FSA written in Prolog [2]. FSA offers determinization, minimization, Epsilon removal and other algorithms, and also supports transducers. Unfortunately, Prolog is not so widely used and the library is outdated. Therefore, the goal of this thesis is to design and implement a new library that allows easy and fast prototyping of advanced algorithms for symbolic automata and symbolic transducers. It should allow easy implementation of new operations as well as adding new types of predicates. It should be used for fast implementation and optimisation of advanced algorithms.

Chapter 2 contains theoretical background for the thesis. Various algorithms for symbolic automata are described in Chapter 3. Efficient algorithms for automata are discussed in Chapter 4. Existing libraries for automata are introduced in Chapter 5. The design of created library is described in Chapter 6. Implementation details of symbolic automata library can be found in Chapter 7. Chapter 8 contains data from the experimental evaluation of created library. Summarization and possibilities for future work can be found in Chapter 9.

Chapter 2

Preliminaries

This chapter contains theoretical background for this thesis. First, languages and classic finite automata will be defined, then predicates and operations over them. After the definition of predicates, symbolic automata and transducers are introduced. Proofs are not given but can be found in the referenced literature [8, 10, 12, 15, 16].

2.1 Languages

Let Σ be an *alphabet* - a finite, nonempty set of symbols. Common examples of alphabets include the binary alpabet ($\Sigma = \{0, 1\}$) or an alphabet of lowercase letters ($\Sigma = \{a, b, ..., z\}$).

A *word* or a *string* w over Σ of *length* n is a finite sequence of symbols $w = a_1 \cdots a_n$, where $\forall 1 \leq i \leq n \ : \ a_i \in \Sigma$. An *empty word* is denoted as ε, such that $\varepsilon \notin \Sigma$ and its length is 0. We define *concatenation* as an associative binary operation on words over Σ represented by the symbol $\cdot$ such that for two words $u = a_1 \cdots a_n$ and $v = b_1 \cdots b_m$ over Σ it holds that $\varepsilon \cdot u = u \cdot \varepsilon = u$ and $u \cdot v = a_1 \cdots a_n b_1 \cdots b_m$. Some strings from binary alphabet $\Sigma = \{0, 1\}$ are for example 00, 110 and their concatenation is 00110.

Σ^* represents a set of all strings over Σ including the empty word. A *language* $L \subseteq \Sigma^*$ is a set of strings where all strings are chosen from Σ^*. A language over binary alphabet is for example $L = \{0, 01, 10, 11, 111\}$.

When $L = \Sigma^*$, then L is called the *universal language* over Σ.

2.2 Finite automata

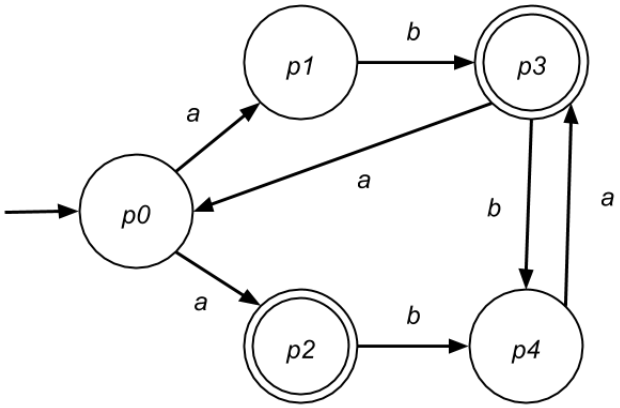
In this section, a definition of finite automata is given and special types of finite automata are described. The section further deals with operations over automata and algorithms for automata processing.

2.2.1 Nondeterministic finite automaton

A *nondeterministic finite automaton* (NFA) is a tuple $A = (\Sigma, Q, I, F, \delta)$, where:

- Σ is a finite alphabet
- Q is a finite set of states
- $I \subseteq Q$ is a nonempty set of initial states

Figure 2.1: Nondeterministic finite automaton



- $F \subseteq Q$ is a set of final states
- δ is a partial function $Q \times \Sigma \to 2^Q$. If $q \in \delta(p, a)$ we use $p \xrightarrow{a} q$ to denote the transition from the state p to the state q with the label a.

An example of NFA A is shown in Figure 2.1. In this case, $\Sigma = \{a, b\}$, $Q = \{p0, p1, p2, p3, p4\}$, $I = \{p0\}$, $F = \{p2, p3\}$, $\delta = \{(p0, a, \{p1, p2\}), (p1, b, \{p3\}), (p2, b, \{p4\}), (p3, a, \{p0\}), (p3, b, \{p4\}), (p4, a, \{p3\})\}$. Note two nondeterministic transitions from $p0$. I.e., it cannot be deterministically decided whether to go to state $p1$ or $p2$ from state $p0$ under a.

2.2.2 Deterministic finite automaton

A *deterministic finite automaton* (DFA) is a special case of an NFA, where $|I| = 1$ and δ is a partial function $Q \times \Sigma \to Q$, i.e, if $\delta(p, a) = q$, then DFA cannot contain another transition where $\delta(p, a) = q'$ such that $q \neq q'$. A DFA is a tuple $A = (\Sigma, Q, I, F, \delta)$, where:

- Σ is an alphabet
- Q is a finite set of states
- $I \subseteq Q$ is a set of initial states where $|I| = 1$
- $F \subseteq Q$ is a set of final states
- $\delta \subseteq Q \times \Sigma \to Q$ is a partial function; we use $p \xrightarrow{a} q$ to denote that $\delta(p, a) = q$

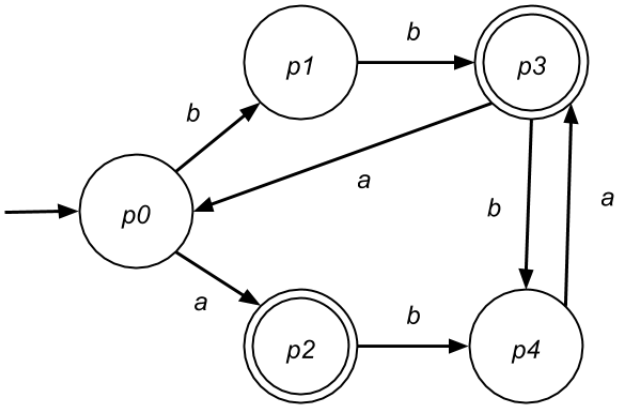
Informally, a DFA must have exactly one initial state and cannot contain more transitions from one state labelled with the same symbol. An example of DFA A is shown in Figure 2.2.

2.2.3 Run of a finite automaton

A *run* of an NFA $A = (\Sigma, Q, I, F, \delta)$ from a state q over a word $w = a_1 \cdots a_n$ is a sequence $r = q_0 \cdots q_n$, where $0 \leq i \leq n$ and $q_i \in Q$ such that $q_0 = q$ and $q_i \xrightarrow{a_{i+1}} q_{i+1} \in \delta$.

The run r is called *accepting* if $q_n \in F$. A word $w \in \Sigma^*$ is called *accepting* if there exists an accepting run from some initial state over w.

Figure 2.2: Deterministic finite automaton



An *unreachable* state q of an NFA $A = (\Sigma, Q, I, F, \delta)$ is a state for which there is no run $r = q_0 \cdots q$ of A over a word $w \in \Sigma^*$ such that $q_0 \in I$. Informally, it is a state that cannot be reached starting from any initial state.

A *useless* or *nonterminating* state q of an NFA $A = (\Sigma, Q, I, F, \delta)$ is a state such that there is no accepting run $r = q \cdots q_n$ of A over a word $w \in \Sigma^*$. Informally, it is a state from which no final state can be reached.

2.2.4 Language of a finite automaton

Consider an NFA $A = (\Sigma, Q, I, F, \delta)$. The *language* of a state $q \in Q$ is defined as

$$L_A(q) = \{w \in \Sigma^* \mid \textit{there exists an accepting run of } A \textit{ from } q \textit{ over } w\}$$

Given a pair of states $p, q \in Q$ of an NFA $A = (\Sigma, Q, I, F, \delta)$, these states are language equivalent if:

$$\forall w \in \Sigma^* : A \textit{ run from } p \textit{ over } w \textit{ is accepting} \Leftrightarrow A \textit{ run from } q \textit{ over } w \textit{ is accepting.}$$

The language of a set of states $R \subseteq Q$ is defined as $L_A(R) = \bigcup_{q \in R} L_A(q)$. The language of an NFA A is defined as $L_A = L_A(I)$.

2.2.5 Complete DFA

DFA $A = (\Sigma, Q_C, I_C, F_C, \delta_C)$ is *complete* iff for any $p \in Q_C$ and every $a \in \Sigma$ exists $q \in Q_C$ such that $p \xrightarrow{a} q \in \delta_C$. Every DFA can be transformed into a complete DFA in two simple steps:

- add a new state to Q, for example $sink \notin Q$, as a nonterminating state
- for every $(q, a) \in Q \times \Sigma$ which is not defined in δ add transition $q \xrightarrow{a} sink$ to δ

2.2.6 Well–specified DFA

$Well - specified$ DFA $A = (\Sigma, Q_C, I_C, F_C, \delta_C)$ is DFA where:

- Q has no unreachable state
- Q has at most one nonterminating state

2.2.7 Minimal DFA

Minimal DFA $A = (\Sigma, Q, I, F, \delta)$ is a complete DFA where:

- there are no unreachable states
- there is at most one nonterminating state
- there are no two language equivalent states

2.2.8 Finite automata operations

In this section, operations over finite automata are described. In the algorithms, *macrostate* Q' denotes a set of states $Q' \subseteq Q$.

Intersection

Intersection of two NFA $A = (\Sigma, Q_A, I_A, F_A, \delta_A)$ and $B = (\Sigma, Q_B, I_B, F_B, \delta_B)$

$$A \cap B = (\Sigma, Q_A \times Q_B, I_A \times I_B, F_A \times F_B, \delta)$$

where δ is defined as

$$\delta = \{(p_1, q_1) \xrightarrow{a} (p_2, q_2) | p_1 \xrightarrow{a} p_2 \in \delta_A \wedge q_1 \xrightarrow{a} q_2 \in \delta_B\}$$

The construction yields an automaton with language $L_{A \cap B} = L_A \cap L_B$

Union

Union of two NFA $A = (\Sigma, Q_A, I_A, F_A, \delta_A)$ and $B = (\Sigma, Q_B, I_B, F_B, \delta_B)$ is defined as

$$A \cup B = (\Sigma, Q_A \cup Q_B, I_A \cup I_B, F_A \cup F_B, \delta_A \cup \delta_B)$$

The construction yields an automaton with language $L_{A \cup B} = L_A \cup L_B$

Determinization

The determinization algorithm transforms any NFA $A = (\Sigma, Q, I, F, \delta)$ into a language equivalent DFA $A_D = (\Sigma, Q_D, I_D, F_D, \delta_D)$. Algorithm 1 determinizing NFA is called *powerset construction*. An NFA may have different runs on a word w that use different transitions and therefore may lead to different states. A DFA must have at most one run on a word w.

In powerset construction algorithm, we create macrostates $Q' \subseteq Q$ of NFA that become states of the resulting DFA. At line 2 of the pseudocode, we initialize a queue W with the first macrostate representing the initial states of the NFA. Then we inspect the macrostates $Q' \in W$ while W is not empty. Intuitively, if at least one $q \in Q'$ is a final state of the NFA, the macrostate Q' will be the final state of the DFA (lines 6–8 of the pseudocode). For every Q' we compute its transition over the symbol a by uniting right-handed sides of the transitions $\delta(q, a)$ for each $q \in Q'$ (line 10 in the pseudocode). If the resulting macrostate Q'' was not yet processed, we add it to the queue W for further inspection (lines 11–13 of the pseudocode).

Algorithm 1: Algorithm for determinization of NFA

Input: NFA $A = (\Sigma, Q, I, F, \delta)$
Output: DFA $A_D = (\Sigma, Q_D, I_D, F_D, \delta_D)$ where $L_D = L_A$

$Q_D, \delta_D, F_D \leftarrow \emptyset$;
$\mathcal{W} = \{I\}$;
while $\mathcal{W} \neq \emptyset$ **do**
    pick Q' from $\mathcal{W}$;
    add Q' to Q_D;
    if $Q' \cap F \neq \emptyset$ **then**
        add Q' to F_D;
    end
    for $a \in \Sigma$ **do**
        $Q'' \leftarrow \bigcup_{q \in Q'} \delta(q, a)$;
        if $Q'' \notin Q_D$ **then**
            add Q'' to $\mathcal{W}$;
        end
        add (Q', a, Q'') to δ_D;
    end
end

Complement

Complement of a complete DFA $A = (\Sigma, Q, I, F, \delta)$ is defined as

$$A_C = (\Sigma, Q, I, Q - F, \delta)$$

The construction yields an automaton with language $L_{A_C} = \Sigma^* - L_A$

Minimization

Minimization transforms DFA $A = (\Sigma, Q, I, F, \delta)$ into a language equivalent minimal DFA. The idea of the transformation is to split states into equivalence classes according to language equivalence relation. The algorithm for this transformation is called *language partition* and can be found in Algorithm 2. After computing language partition we merge the states of each equivalence class into a macrostate. If all states in the equivalence class are final states of the original DFA, this macrostate is final in the resulting minimal DFA. There is a transition (B, a, B') from macrostate B to B' if there exists a transition $\delta(q, a) = q'$, such that $q \in B$, $q' \in B'$.

2.3 Regular Languages

A language L is *regular* if there exists an NFA $A = (\Sigma, Q, I, F, \delta)$ that $L = L_A$.

2.3.1 Closure Properties

Regular languages are closed under an operation if the operation on some regular languages always results in a regular language.

Let L_1 and L_2 b regular languages. Closure properties of these languages are following:

Algorithm 2: Algorithm for language partition

Input: DFA $A = (\Sigma, Q, I, F, \delta)$
Output: language partition P

```
if F = ∅ or Q − F = ∅ then
    return {Q};
else
    P ← {F, Q − F};
end
W ← {(a, min{F, Q − f}}|a ∈ Σ;
while W ≠ ∅ do
    pick (a, B′) from W;
    for B ∈ P split by (a, B′) do
        replace B by B0 and B1 in P;
        for b ∈ Σ do
            if (b, B) ∈ W then
                replace (b, B) by (b, B0) and (b, B1) in W;
            else
                add (b, min{B0, B1}) to W;
            end
        end
    end
end
```

- Union: $L = L_1 \cup L_2$.
- Intersection: $L = L_1 \cap L_2$.
- Complement: $L = \overline{L_1}$.
- Difference: $L = L_1 - L_2$.
- Reversal: $L = \{a_1 \dots a_n \in \Sigma^* \mid y = a_n \dots a_1 \in L\}$.
- Concatenation: $L \cdot K = \{x \cdot y \mid x \in L \wedge y \in K\}$.

The first three operations (union, intersection and complement) can be done using finite automata representation for given regular language. A description of these operations over finite automata can be found in Section 2.2.8. The detailed descriptions, proofs and algorithms for the other operations can be found in the referenced literature [8, 12].

2.3.2 Decidability

A problem about regular language is decidable if there is such an algorithm that for every regular language answers the problem *yes* or *no*.

Decidable problems for regular languages are:

- *Emptiness* problem: Is language L empty?
- *Membership* problem: Does a particular string w belong to language L?

- *Equivalence* problem: Does language L_1 describe the same language as language L_2?
- *Infiniteness* problem: Is language L infinite?
- *Finiteness* problem: Is language L finite?
- *Inclusion* problem: Is $L_1 \subseteq L_2$?
- *Universality* problem: Is $L = \Sigma^*$?

2.4 Finite transducer

Finite transducers differ from finite automata in transition labels. While finite automata labels contain only one input symbol, finite transducers have input and also output symbols. Therefore, transducers have two alphabets — one for input symbols and one for output symbols. Finite transducers can be informally described as translators which for an input symbol generate an output symbol.

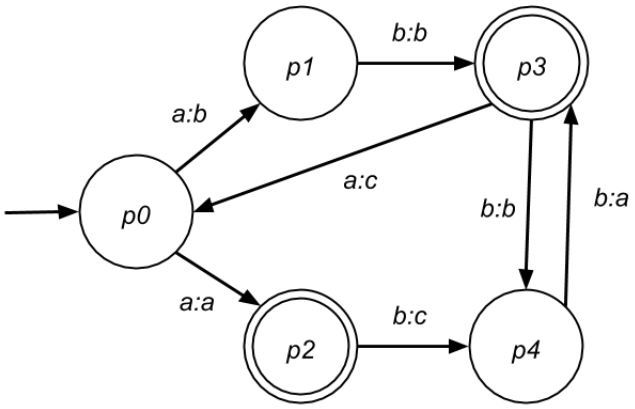
A *nondeterministic finite transducer* (NFT) is a tuple $T = (\Sigma, \Omega, Q, I, F, \delta)$, where:

- Σ is an input alphabet
- Ω is an output alphabet
- Q is a finite set of states
- $I \subseteq Q$ is a nonempty set of initial states
- $F \subseteq Q$ is a set of final states
- $\delta \subseteq Q \times (\Sigma : \Omega) \times Q$ is the transition relation. We use $p \xrightarrow{a:b} q$ to denote that there is a transition from the state p to the state q under the input symbol a and the output symbol b.

An example of a nondeterministic finite transducer is shown in Figure 2.3.

A *deterministic finite transducer* (DFT) must have exactly one initial state and cannot contain more transitions from one state under the same input symbol.

Figure 2.3: Nondeterministic finite transducer



2.4.1 Translation of a finite transducer

A *configuration* of a finite state transducer $M = (\Sigma, \Omega, Q, I, F, \delta)$ is a string vpu where $p \in Q$, u in Σ^* and $v \in \Omega^*$.

Let $vpxu$ and $vyqu$ be two configurations of transducer $M = (\Sigma, \Omega, Q, I, F, \delta)$ where $p, q \in Q$, x, u in Σ^* and $v, y \in \Omega^*$. The transducer M makes a *move* from $vpxu$ to $vyqu$ according to transition r written as $vpxu \vdash vyqu[r]$ or simply $vpxu \vdash vyqu$. Informally, the transition r changes the current state from p to q and rewrites the input symbol x to output symbol y. We use $\vdash^*$ to denote a sequence of consecutive moves.

A translation $T(M)$ of a finite transducer is:

$$T(M) = \{(x, y) : sx \vdash^* yf, x \in \Sigma^*, y \in \Omega^*, f \in F\}$$

An input language corresponding to $T(M)$ is:

$$L_I(M) = \{x : (x, y) \in T(M) \ for\ some\ y \in \Omega^*\}$$

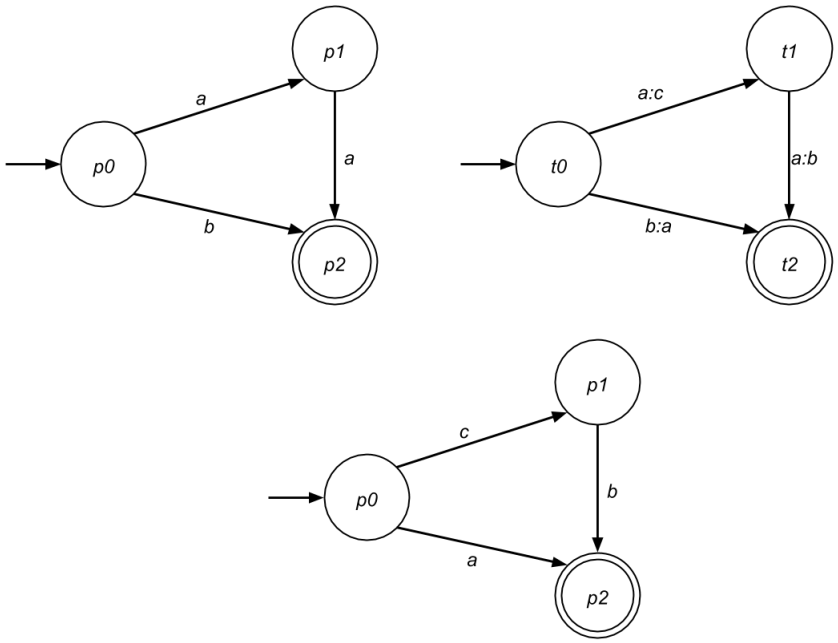
An output language corresponding to $T(M)$ is:

$$L_O(M) = \{y : (x, y) \in T(M) \ for\ some\ x \in \Sigma^*\}$$

2.4.2 Application of a finite transducer

Besides input string, a finite transducer can be applied on a finite automaton. In this operation, instead of changing an input symbol to an output symbol, the transducer changes the symbols in the automata transitions. An example of the application is in Figure 2.4. The Figure shows an input automaton and a transducer, below them is the output automaton.

Figure 2.4: Application of a finite transducer



2.5 Predicates

A *predicate* π is a formula representing a subset of Σ. Π is a set of predicates such that for each element $a \in \Sigma, \exists \pi \in \Pi : \pi$ *represents* $\{a\}$ and Π is effectively closed under Boolean operations.

We will further use predicates *in* and *not_in* inspired by a Prolog library FSA [2]. The semantics of these predicates is:

- $in\{a_1, a_2, \ldots, a_i\}$ represents a subset $\{a_1, a_2, \ldots, a_i\} \in \Sigma$
- $not\_in\{a_1, a_2, \ldots, a_i\}$ represents a subset $\Sigma - \{a_1, a_2, \ldots, a_i\}$

2.5.1 Operations over predicates

Predicates must support conjunction, disjunction and complement since these operations are required for the algorithms described in Chapter 3. The semantics of conjunction and disjunction of *in* and *not_in* predicates can be found in Table 2.1 and Table 2.2 and the complement in Table 2.3.

Table 2.1: Conjunction of predicates

x	**y**	**x** $\wedge$ **y**
$in\{a_0, a_1, \ldots, a_n\}$	$in\{b_0, b_1, \ldots, b_m\}$	$in\{\{a_0, a_1, \ldots, a_n\} \cap \{b_0, b_1, \ldots, b_m\}\}$
$in\{a_0, a_1, \ldots, a_n\}$	$not\_in\{b_0, b_1, \ldots, b_m\}$	$in\{\{a_0, a_1, \ldots, a_n\} - \{b_0, b_1, \ldots, b_m\}\}$
$not\_in\{a_0, a_1, \ldots, a_n\}$	$in\{b_0, b_1, \ldots, b_m\}$	$in\{\{b_0, b_1, \ldots, b_m\} - \{a_0, a_1, \ldots, a_n\}\}$
$not\_in\{a_0, a_1, \ldots, a_n\}$	$not\_in\{b_0, b_1, \ldots, b_m\}$	$not\_in\{\{a_0, a_1, \ldots, a_n\} \cup \{b_0, b_1, \ldots, b_m\}\}$

Table 2.2: Disjunction of predicates

x	**y**	**x** $\vee$ **y**
$in\{a_0, a_1, \ldots, a_n\}$	$in\{b_0, b_1, \ldots, b_m\}$	$in\{\{a_0, a_1, \ldots, a_n\} \cup \{b_0, b_1, \ldots, b_m\}\}$
$in\{a_0, a_1, \ldots, a_n\}$	$not\_in\{b_0, b_1, \ldots, b_m\}$	$not\_in\{\{b_0, b_1, \ldots, b_m\} - \{a_0, a_1, \ldots, a_n\}\}$
$not\_in\{a_0, a_1, \ldots, a_n\}$	$in\{b_0, b_1, \ldots, b_m\}$	$not\_in\{\{a_0, a_1, \ldots, a_n\} - \{b_0, b_1, \ldots, b_m\}\}$
$not\_in\{a_0, a_1, \ldots, a_n\}$	$not\_in\{b_0, b_1, \ldots, b_m\}$	$not\_in\{\{a_0, a_1, \ldots, a_n\} \cap \{b_0, b_1, \ldots, b_m\}\}$

Table 2.3: Complement of predicates

x	$\neg$ **x**
$in\{a_0, a_1, \ldots, a_n\}$	$not\_in\{a_0, a_1, \ldots, a_n\}$
$not\_in\{a_0, a_1, \ldots, a_n\}$	$in\{a_0, a_1, \ldots, a_n\}$

2.6 Symbolic automata

A *symbolic automaton* A is a tuple $A = (\Sigma, Q, I, F, \Pi, \delta)$, where:

- Σ is an alphabet
- Q is a finite set of states

- $I \subseteq Q$ is a nonempty set of initial states
- $F \subseteq Q$ is a set of final states
- Π is a set of predicates over Σ
- $\delta \subseteq Q \times (\Pi \cup \{\varepsilon\}) \times Q$ is the transition relation.

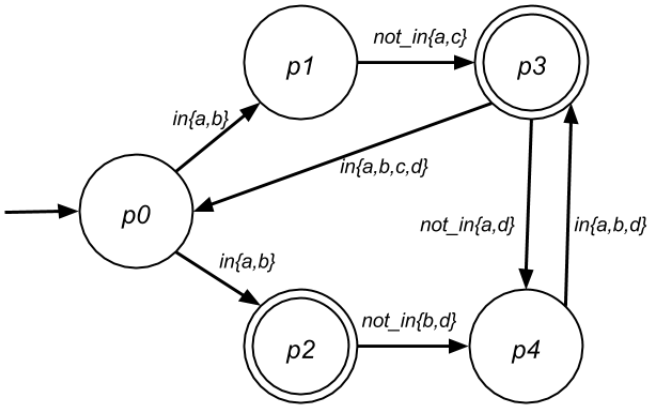
An example of SA A with predicates *in* and *not_in* introduced in Section 2.5 is shown in Figure 2.5.

Every SA with a finite alphabet can be transformed into a NFA in two steps:

- remove Π
- for each transition $p \xrightarrow{\pi} q \in \delta$:
  - create a transition $p \xrightarrow{a} q$ for every $a \in \pi$

Since every SA can be transformed into NFA, the closure and decidability properties of symbolic automata are the same as finite automata (described in Section 2.3.1 and Section 2.3.2). Also, every operation applicable on finite automata can be used on symbolic automata after transformation to a finite automaton. The transformation can be usually avoided because most of the algorithms for finite automata can be modified to a variation for symbolic automata. The modified algorithms will be described in Chapter 3.

Figure 2.5: Symbolic automaton



2.7 Symbolic transducers

A *symbolic transducer* A is a tuple $A = (\Sigma, \Omega, Q, I, F, \Pi, \delta)$, where:

- Σ is an input alphabet
- Ω is an output alphabet
- Q is a finite set of states
- $I \subseteq Q$ is a nonempty set of initial states

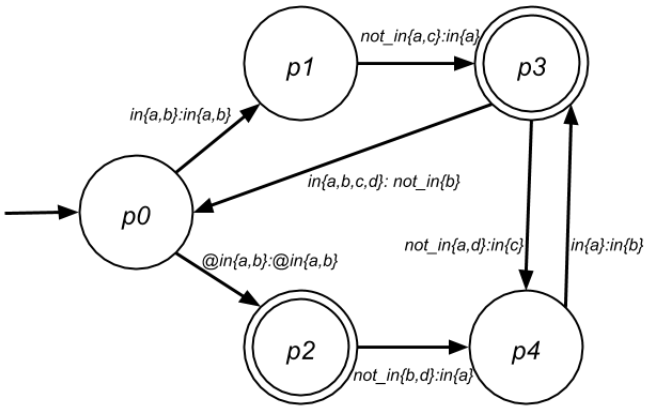
- $F \subseteq Q$ is a set of final states
- Π is a set of predicates over Σ
- $\delta \subseteq Q \times (\Pi \cup \{\varepsilon\}) \times (\Pi \cup \{\varepsilon\}) \times Q$ is the transition relation.

A special case of a transition is *identity*. Identity takes a symbol from the set represented by the input predicate and copies it on the output. In the case of *in* and *not_in* predicates it is denoted by @ in front of the predicate.

An example of a symbolic transducer is shown in Figure 2.6. Transition from $p0$ to $p1$, $p0 \xrightarrow{in\{a,b\}:in\{a,b\}} p1$ would be represented as these 4 transitions in classic transducer: $p0 \xrightarrow{a:a} p1$, $p0 \xrightarrow{a:b} p1$, $p0 \xrightarrow{b:a} p1$, $p0 \xrightarrow{b:b} p1$. The syntax of identity can be seen in the transition from $p0$ to $p2$: $p0 \xrightarrow{@in\{a,b\}:@in\{a,b\}} p2$. In the classic transducer, it would be represented by the transitions $p0 \xrightarrow{a:a} p2$ and $p0 \xrightarrow{b:b} p2$.

Symbolic transducers allow more concise representation on finite transducers. Since symbolic transducers can be transformed to classic finite transducers by creating transitions for each pair of symbols represented by input and output predicates, most operations applicable on finite transducers can be applied on symbolic transducers as well. The transformation can be avoided, because algorithms can usually be generalised to work on symbolic transducers as in the case of symbolic automata. The modifications of some of the algorithms is discussed in Chapter 3.

Figure 2.6: Symbolic finite transducer



Chapter 3

Algorithms for symbolic automata and transducers

This chapter describes various algorithms for symbolic automata and transducers. The most of the algorithms are generalised to work with predicates instead of letters [6, 16].

Symbolic automata are more general variant of finite automata. However, the algorithms for finite automata are also applicable on symbolic automata. In the worst case, the algorithms would expand symbolic automata to ordinary finite automata then perform the desired operation and transform the resulting automata back to symbolic form. Fortunately, this process is not necessary for most of the algorithms. As will be shown, most of the algorithms can be generalised and work directly with symbolic automata. The same holds for symbolic transducers.

3.1 Intersection

In the case of classic finite automata, the intersection of two NFA $A = (\Sigma, Q_A, I_A, F_A, \delta_A)$ and $B = (\Sigma, Q_B, I_B, F_B, \delta_B)$ is defined as:

$$A \cap B = (\Sigma, Q_A \times Q_B, I_A \times I_B, F_A \times F_B, \delta)$$

where δ is defined as:

$$\delta = \{(p_1, q_1) \xrightarrow{a} (p_2, q_2) | p_1 \xrightarrow{a} p_2 \in \delta_A \wedge q_1 \xrightarrow{a} q_2 \in \delta_B\}$$

The same approach can be used for symbolic automata, but instead of requiring the symbol $a \in \Sigma$ to be the same in both of the automata, we consider a conjunction of the corresponding predicates in A and B, as shown below. For symbolic transducers we consider a component-wise conjunction of both the input predicates and the output predicates.

$$\delta = \{(p_1, q_1) \xrightarrow{\pi_A \wedge \pi_B} (p_2, q_2) | p_1 \xrightarrow{\pi_A} p_2 \in \delta_A \wedge q_1 \xrightarrow{\pi_B} q_2 \in \delta_B\}$$

3.2 Union

Union of symbolic automata can be computed the same way as for classical finite automata which was described in Section 2.2.8.

$$A \cup B = (\Sigma, Q_A \cup Q_B, I_A \cup I_B, F_A \cup F_B, \Pi_A \cup \Pi_B, \Pi_A \cup \Pi_B, \delta_A \cup \delta_B)$$

3.3 Determinization

In determinization algorithm for classic finite automata described in Section 2.2.8, we use macrostates. Each macrostate is a set in the resulting deterministic machine. To compute transitions leaving a macrostate Q', we unite right-handed sides of the transitions $\delta(q, a)$ for each $q \in Q'$.

In the case of symbolic automata, predicates of transitions leaving the given macrostate might represent non-disjoint sets. This situation cannot happen in deterministic automata and thus we must create disjoint predicates. E.g., for transitions labelled with predicates π_1 and π_2, we the transitions labelled with following predicates: $\pi_1 \wedge \overline{\pi_2}$, $\pi_1 \wedge \pi_2$, $\overline{\pi_1} \wedge \pi_2$. This process is called *getting exclusive predicates* for a macrostate and can be found in Algorithm 3.

Algorithm 3: Getting exclusive predicates for a macrostate

Input: NSA $A = (\Sigma, Q, I, F, \Pi, \delta)$, macrostate $Q' \in 2^Q$
Output: Exclusive predicates for Q'

```
1 Π_e ← ∅;
2 for each q ∈ Q' do
3 |   Π' = {π|(q, π) ∈ δ};
4 |   C ← disjoint sets representing Π';
5 |   Π_e = Π_e ∪ C;
6 end
7 return Π_e
```

After computing the exclusive predicates, we compute transitions leaving a macrostate Q' by uniting right-handed sides of the transitions $\delta(q, \pi)$ for each $q \in Q'$ and each $\pi \in \Pi_e$ (lines 10-11 in Algorithm 4).

3.4 Minimization

In Hopcroft's minimization algorithm [11], we repeatedly refine subsets of states by refining sets of states to equivalence classes of language equivalence relation. The algorithm ends when such a pair no longer exists. The number of states is minimal in the resulting automaton. The algorithm for minimization of classic finite automata was given in Section 2.2.8.

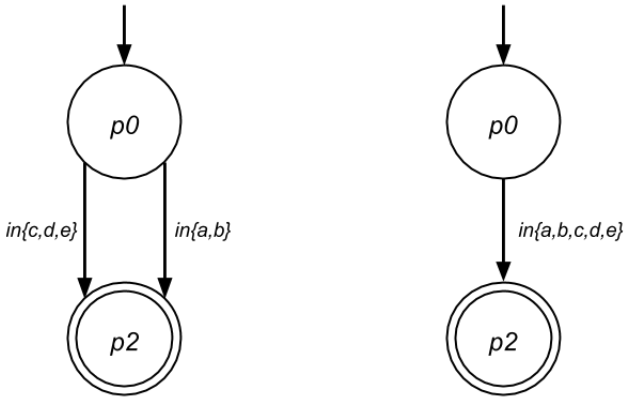
However, in the case of symbolic auomata, the resulting automaton might not be minimal in the number of transitions, because the same transition can be expressed in multiple ways. E.g., a transition $p0 \xrightarrow{in\{a,b,c\}} p1$ could also be represented by two transitions: $p0 \xrightarrow{in\{a,b\}} p1$ and $p0 \xrightarrow{in\{c\}} p1$. Therefore, as the final step of minimization, we must perform a *cleanup* that joins all transitions from state p to state q into one, which is done by uniting all transition predicates leading from p to q. After this, the resulting automaton will be minimal in the number of transitions. An example of cleanup is shown in Figure 3.1. On the left are the transitions before cleanup and on the right after cleanup.

Algorithm 4: Algorithm for determinization of SA

Input: NSA $A = (\Sigma, Q, I, F, \Pi, \delta)$
Output: DSA $A_D = (\Sigma, Q_D, I_D, F_D, \Pi, \delta_D)$ where $L_D = L_A$

$Q_D, \delta_D, F_D \leftarrow \emptyset$;
$\mathcal{W} = \{I\}$;
while $\mathcal{W} \neq \emptyset$ **do**
    pick Q' from $\mathcal{W}$;
    add Q' to Q_D;
    if $Q' \cap F \neq \emptyset$ **then**
        add Q' to F_D;
    end
    $\Pi_e \leftarrow$ get exclusive predicates for Q';
    for $\pi \in \Pi_e$ **do**
        $Q'' \leftarrow \bigcup_{q \in Q'} \delta(q, \pi)$;
        if $Q'' \notin Q_D$ **then**
            add Q'' to $\mathcal{W}$;
        end
        add (Q', π, Q'') to δ_D;
    end
end

Figure 3.1: Minimization cleanup



3.5 Optimization

Previous algorithms requiring non-overlapping predicates overlap produce a lot of their combinations. Therefore the resulting automata might significantly grow in the number of transitions. This can be avoided by using simple optimisations.

3.5.1 Satisfiability

A predicate P is $satisfiable$, when it does not represent an empty set. Unsatisfiable transitions can be removed from the automaton because they will never be used.

An example of unsatisfiable predicate is $in\{a, b\} \wedge in\{c, d\}$. A transition labelled with this predicate will never be used because a symbol that is in set $\{a, b\}$ and also in set $\{c, d\}$ does not exist. Thus, this transition can be removed from the automaton without changing the language of an automaton.

3.5.2 Removing useless states

This optimisation is not dependent on the use of predicates and can be used in classic finite automata as well. $Useless\ states$ are the states, from which no final state can be reached. All useless states can be removed from the automaton without changing the accepted language. All transitions $p \xrightarrow{a} q$ where p is a useless state can also be removed. The algorithm for removing useless states is the same as in the case of classic finite automata.

Chapter 4

Efficient algorithms for automata and transducers

In this chapter we describe efficient algorithms for automata manipulation. This thesis covers only the basics, more complex proofs and theories can be found in the referenced literature [9, 10, 14].

In many finite and symbolic automata operations, it is efficient to work with a minimised version of the automaton. However, the naive minimisation algorithm determinizes the automaton first. Determinization is in the worst cases inefficient because the size of the automaton can exponentially increase. To avoid this increase, NFA can use equivalence or simulation relations for states reduction. A reduction based on simulations usually yields an automaton smaller than minimal DFA but the resulting automaton is not deterministic.

4.1 Reduction and equivalence

Let $A = (\Sigma, Q, I, F, \Pi, \delta)$ be an NFA. Equivalence relation $\equiv_R \subseteq Q \times Q$ is defined as:

- $\equiv_R \cap (F \times (Q - F)) = \emptyset$
- for any $p, q \in Q, a \in \Sigma, (p \equiv_R q \Rightarrow (\forall q' \in \delta(q, a), \exists p' \in \delta(p, a), p' \equiv_R q' \; and \; \forall p' \in \delta(p, a), \exists q' \in \delta(q, a), q' \equiv_R p'))$

The first condition simply means that a final state cannot be equivalent to a nonfinal state. The second condition means that two states p and q are equivalent when for every symbol a such that transition $p \xrightarrow{a} p' \in \delta$ a transition $q \xrightarrow{a} q' \in \delta$ must exist and the states p' and q' must also be equivalent.

Symmetrically, a relation $\equiv_L$ can be defined over an reversed automaton. By reversed automaton is meant an automaton, in which transition have been reversed by the rule $q \in \delta_r(p, a)$ if $p \in \delta(q, a)$.

An automaton can be reduced using both equivalencies, but the automaton reduced by $\equiv_R$ and the automaton reduced by $\equiv_L$ do not need to be equivalent.

Implementing reduction directly by the definition would lead to exponential rise of used memory space because computing equivalence for one state leads to computing equivalence for all the following ones. The computation would recursively check all the following states until it reaches a state with no transitions or finds a counterexample. This strategy is not computationally possible for big automata. The more efficient method could be checking

the equivalence in the reversed order, by checking the predecessors of a state which may eliminate the recursion.

After computing equivalence, the algorithm to reduce the automaton A is trivial: it simply merges all state in the same equivalence class into one and modifies the transitions accordingly.

4.2 Reduction and simulations

A stronger reduction can be efficiently obtained by using simulations instead of equivalence. The definition of simulation $\preceq_R \subseteq Q \times Q$ is similar to equivalence:

- $\preceq_R \cap (F \times (Q - F)) = \emptyset$
- for any $p, q \in Q, a \in \Sigma, (p \preceq_R q \Rightarrow \forall q' \in \delta(q, a), \exists p' \in \delta(p, a), p' \preceq_R q')$

As in the case with equivalencies, $\preceq_L$ can be created using the reversed automaton. If $p \preceq_R q$, then $L_R(p) \subseteq L_R(q)$ and if $p \preceq_L q$ then $L_L(p) \subseteq L_L(q)$.

The reduction using simulations is more complicated than the reduction using equivalencies. We can merge two states p and q as soon as any of the conditions is met:

1. $p \preceq_R q$ and $q \preceq_R p$
2. $p \preceq_L q$ and $q \preceq_L p$
3. $p \preceq_R q$ and $p \preceq_L q$

However, after merging the states according to conditions 1 or 2, relations $\preceq_R$ and $\preceq_L$ must be updated so that their relation with the languages L_R and L_L is preserved. For instance, if merged state of p and q is denoted q, we must remove from $\preceq_R$ any pairs (q, s) for which $p \not\preceq_R s$. Merging according to condition 3 does not require any update.

Pseudocode for efficient computation of simulations is given in Algorithm 5. The algorithm works correctly only on the complete NFAs. In this algorithm, $card(n)$ denotes the number of items in a set n. The result of this algorithm is $\not\preceq_R$, which is a complement of $\preceq_R$. $\not\preceq_R$ is defined as:

- $(F \times (Q - F)) \subseteq \not\preceq_R$
- for any $p, q \in Q, a \in \Sigma, (\exists p' \in \delta(p, a), \forall q' \in \delta(q, a), p' \not\preceq_R q' \Rightarrow p \not\preceq_R q)$

Since $\not\preceq_R$ is a complement of $\preceq_R$, this algorithm returns all pair of states (p, q) in which q does not simulate p. The same algorithm can be used for computing $\not\preceq_L$ when it is applied on the reversed automaton.

Further in this thesis, $\preceq$ is used as a shorthand for $\preceq_R$.

4.3 Antichains

The textbook approach to language inclusion checking requires both automata to be determinized first. This approach is inefficient when working with big alphabets and automata. The antichain algorithm allows us to skip the complete determinization. While checking if $L_A \subseteq L_B$, the automaton A is not determinized at all and the automaton B is determinized

Algorithm 5: Algorithm for simulations computation. Algorithm is taken from [14]

Input: complete NFA $A = (\Sigma, Q, I, F, \delta)$
Output: $\not\preceq_R$

```
for each q ∈ Q and a ∈ Σ do
    compute δ^r(q, a) as an linked list;
    compute card(δ(q, a));
end
initialize all N(a)s with 0s;
ω := ∅;
C := NEW_QUEUE();
for each f ∈ F do
    for each q ∈ Q − F do
        ω := ω ∪ {(f, q)};
        ENQUEUE(C, (f, j));
    end
end
while C ≠ ∅ do
    (i, j) := DEQUEUE(C) ;
    for each a ∈ Σ do
        for each k ∈ δ^r(j, a) do
            N(a)_ik := N(a)_ik + 1;
            if N(a)_ik == card(δ(k, a)) then
                for l ∈ δ^r(i, a) do
                    if (l, k) ∉ ω then
                        ω := ω ∪ {(l, k)};
                        ENQUEUE(C, (l, k));
                    end
                end
            end
        end
    end
end
```

gradually. If $L_A \not\subseteq L_B$, the algorithm stops when finding the first counterexample, so the automaton B is not completely determinized.

We define an antichain and some others terms before describing the algorithm itself. Given a partially ordered set Y, an *antichain* is a set $X \subseteq Y$ such that all elements of X are incomparable. *Macrostate* is defined as a subset of states from Q. For two macrostates P and R of a NFA is $R \preceq^{\forall\exists} P$ shorthand for $\forall r \in R \,.\, \exists p \in P : r \preceq p$. A product state (p, P) of a NFA $A \cap B_{det}$ is witness, if p is final in automaton A and P is not final in automaton B_{det}.

The antichains algorithm [5] described in pseudocode in Algorithm 6 can be used for language inclusion checking of finite automata. The antichains algorithm tries to find a final state of the product automaton $A \cap \overline{B_{det}}$ while not exploring the states when not necessary. The algorithm explores pairs (p, P) where $p \in Q_A$ and $P \subseteq Q_{B_{det}}$. The automaton B is gradually determinized while constructing $Post(p, P) := \{(p', P') \mid \exists a \in \Sigma : p \xrightarrow{a} p' \in \delta_A, P' = \{p'' \in Q_B \mid \exists p''' \in P : p''' \xrightarrow{a} p'' \in \delta_B\}\}$.

The algorithm derives the new states from the product automaton transitions and inserts them to the set $Next$ for further processing (line 15). Once a product state from $Next$ is processed it is moved to the set of checked pairs $Processed$ (line 7). $Next$ and $Processed$ keep only minimal elements with respect to the ordering given by $(r, R) \sqsubseteq (p, P)$ iff $r = p \wedge R \subseteq P$. If there is a pair (p, P) generated and there is $(r, R) \in Next \cup Processed$ such that $(r, R) \sqsubseteq (p, P)$, we can skip (p, P) and not insert it to $Next$ for further search (lines 13–16).

An improvement of the antichains algorithm is based on simulations. We can stop the search for a pair (p, P) if one of the following conditions is met:

- there exists some already visited pair $(r, R) \in Next \cup Processed$ such that $p \preceq r \wedge R \preceq^{\forall\exists} P$
- there is $p' \in P$ such that $p \preceq p'$

This optimisation is at the lines 12–15 in the pseudo code. The basic explanation is that if the algorithm encounters a macrostate R which is a superset of already checked macrostate P, there is no need to continue the search for this one. If no counterexample was found in the macrostate P, then it cannot be found in R since R is bigger and widens the number of accepted words.

Another optimisation is based on a different principle. It comes from observation that $L(A)(P) = L(A)(P \backslash \{p_1\})$ if there is some $p_2 \in P$ such that $p_1 \preceq p_2$ and $p_1 \neq p_2$. Since P and $P \backslash \{p_1\}$ have the same language, if a word is not accepted from P, it is not accepted from $P \backslash \{p_1\}$ either. Also, if all words from Σ^* are accepted from P, they are also accepted from $P \backslash \{p_1\}$. Therefore, it is safe to replace the macrostate P with macrostate $P \backslash \{p_1\}$. This optimisation is applied by the function $Minimize$ at the lines 5 and 8 in the pseudocode.

4.4 Antichains and simulations for symbolic automata

The implementation of antichains algorithm for symbolic automata is similar than for finite automata, described in Algorithm 6. Although the main part of the algorithm stays the same, some alterations must be made to adapt it to predicates. This section contains only fragments of code representing the alterations since both antichains and simulations algorithms were already described in Section 4.2 and Section 4.3.

The following snippet shows change in simulations computation introduced in Algorithm 5. Since the labels in symbolic automata represent sets of symbols, these computa-

Algorithm 6: Language inclusion checking with antichains and simulations. Algorithm is taken from [13]

Input: NFAs $A = (\Sigma, Q_A, I_A, F_A, \delta_A)$, $B = (\Sigma, Q_B, I_B, F_B, \delta_B)$
A relation $\preceq$ over $A \cup B$ that imply language inclusion.
Output: TRUE if $L_A \subseteq L_B$. Otherwise FALSE.

```
1  if there is a witness product-state in {(i, I_B) | i ∈ I_A} then
2  |  return FALSE;
3  end
4  Processed:=∅;
5  Next:= {(s, Minimize(I_B)) | s ∈ I_A};
6  while Next ≠ ∅ do
7  |  Pick and remove a product-state (r, R) from Next and move it to Processed;
8  |  foreach (p, P) ∈ {(r', Minimize(R')) | (r', R') ∈ Post(r, R)} do
9  |  |  if (p, P) is a witness product-state then
10 |  |  |  return FALSE;
11 |  |  else
12 |  |  |  if ∄p' ∈ P s.t. p ⪯ p' then
13 |  |  |  |  if ∄(x, X) ∈ Processed ∪ Next s.t. p ⪯ x ∧ X ⪯^{∀∃} P then
14 |  |  |  |  |  Remove all (x, X) from Processed ∪ Next s.t. x ⪯ p ∧ P ⪯^{∀∃} X;
15 |  |  |  |  |  Add (p, P) to Next;
16 |  |  |  |  end
17 |  |  |  end
18 |  |  end
19 |  end
20 end
21 return TRUE;
```

tions are implemented by looping through all alphabet symbols and checking if they belong to a predicate. The pseudocode for this alternation can be seen in the fragment of code in Algorithm 7. This alteration is used each time where the original algorithm loops through $a \in \Sigma$ — lines 1 and 16 of Algorithm 5.

Algorithm 7: Transformation of simulations algorithm for symbolic automata

```
1 for a ∈ Σ do
2 |   for π ∈ δ(q, π) do
3 |   |   if a ∈ π then
4 |   |   |   process the transition;
5 |   |   end
6 |   end
7 end
```

The same principle is reused in computation of *Post* over symbolic automata as can be seen in Algorithm 8. We check for each symbol of alphabet whether there are transitions from both states representing the sets of symbols including a.

Algorithm 8: Computing *Post* over symbolic automata

```
1 for a ∈ Σ do
2 |   for π ∈ δ(q, π) do
3 |   |   if a ∈ π then
4 |   |   |   Post = Post ∪ δ(q, π);
5 |   |   end
6 |   end
7 end
```

The used modifications of antichains algorithm for symbolic automata yield correct results, but they are not efficient because they perform implicit transformation to letter automata. Further optimisation of these functions specifically for symbolic automata can make the library created in this thesis faster and are the subjects for further research.

Chapter 5

Existing Libraries for Automata

This chapter contains a brief description of some of the existing libraries that deal with finite or symbolic automata. For every chosen library, implementation language, types of supported automata and other details are given. The real number of automata libraries is bigger, but the inspection of all of them is over the scope of this thesis. Therefore, only the libraries that are closely related to this thesis (by their design or used concept) are mentioned.

5.1 AutomataDotNet

AutomataDotNet [17] is a library by Margus Veanes written in .NET framework in C#. It supports algorithms for regular expressions, automata, and transducers. AutomataDotNet can work with symbolic automata where characters are replaced with character predicates. It also allows to use SMT solver as a plugin. Some of the high efficient algorithms for automata manipulation are implemented, e.g. simulations used for automata reduction.

AutomataDotNet can handle many types of automata from classic finite automata and transducers to tree automata. A big advantage is a possibility to use an SMT solver for predicates. Unfortunately, the library is complex and many modules, their connections and interactions have to be studied for extending it. This is the reason, why the library is not easily modifiable and thus is not suitable for fast prototyping.

5.2 symbolicautomata

Symbolicautomata [7] is a library by Loris d'Antoni written in Java. The library can handle symbolic automata and algorithms over them such as intersection, union, equivalence and minimization. The predicates supported by symbolicautomata by default are character intervals. For example $[a–z]$ represents every symbol in the interval from a to z. It includes support for symbolic pushdown automata and symbolic transducers. Furthermore, some of the high efficient algorithms for automata manipulation such as simulations are implemented.

Due to complexity of the library, the learning curve of this library is high and it is not suitable for easy and fast prototyping of new algorithms.

5.3 VATA

Libvata [4, 18] is a highly optimised non-deterministic finite tree automata library implemented in C++. It uses the most advanced techniques for automata. VATA can deal with classic finite automata as well as both explicit and semi-symbolic encoding of tree automata.

VATA has a modular design so the various encodings for automata can be easily added as long as they respect the defined interface. Different encodings may implement different sets of operations. Generally, VATA offers basic operations, such as union and intersection, and also more complex ones, such as reduction based on simulation or language inclusion checking with antichains optimisation. For some operations, such as language inclusion, different versions of algorithms are available.

VATA lacks transducers, moreover C++ is not really a good language for fast prototyping.

5.4 FAdo

FAdo [1] is a Python library focusing on finite automata and other models of computation. Because this library can work with both regular expressions and automata, it offers only basic algorithms. It can perform conversions between nondeterministic and deterministic automata, as well as conversion between automata and regular expression. FAdo can also transfer automata to simple graphic representations.

Regular expressions, DFA and NFA are implemented as Python classes. Transducers are also available. Elementary regular languages operations as union, intersection, concatenation, complementation and reversion are implemented for each class. For operations like determinization and minimalization, it is possible to choose from more methods, such as Moore, Hopcroft, and some incremental algorithms. Some of these algorithms are implemented in C for higher efficiency.

The biggest advantage of FAdo library is implementation in Python. Python is language that is easy to use and therefore has the potential to be used for fast prototyping of new efficient algorithms. Unfortunately, FAdo library is still immature, more in the state of a prototype, and not very well documented. Extending this library to support symbolic automata would be hard because of the lack of good documentation of its classes, interfaces and inner communication.

5.5 FSA

FSA [2] is a fast library written in Prolog that can handle classic finite automata and transducers as well as symbolic. Predicates *in* and *not_in* which are used in this thesis and will be implemented in the created library are inspired by FSA.

FSA offers determinization, minimization, Epsilon removal and other algorithms. It also supports transducers. Unfortunately, this library is outdated. Also, Prolog is not widely used and adaptation of the library in the community would be hard.

Chapter 6

Design

This chapter contains a description of the library design. The library modules and their connections are discussed.

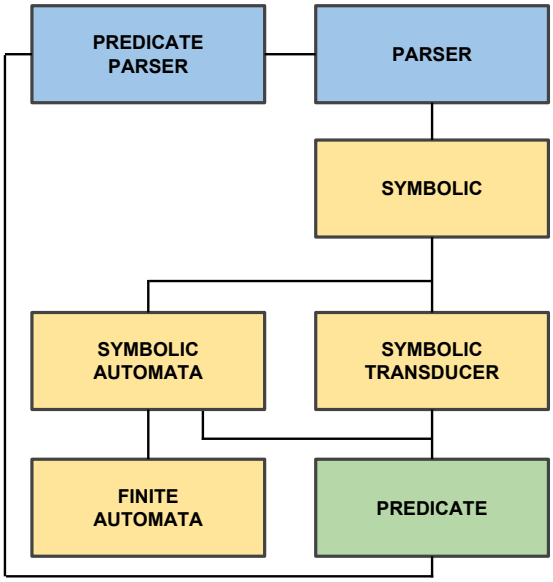
The library is designed to allow easy and intuitive manipulation with symbolic automata and transducers. The library will provide explicit and also symbolic encoding of finite automata. Basic operations should be available for each supported type of automata. Additionally, some state-of-the-art algorithms for finite automata should be implemented. Adding new operations should be straightforward. None of the operations should be dependent on a specific predicate type. Adding new types of predicates should be allowed as long as they respect a defined interface.

6.1 Design of the library

The library consists of multiple modules, as is shown in Figure 6.1. The lines in the Figure signifies that the connected modules interact with each other.

- `Parser` is used for automata and transducer parsing.
- `Predicate parser` is called when `Parser` reaches a predicate. `Predicate parser` should parse the predicate and transform it into a `Predicate` object. The object is then returned and `Parser` stores it in the automaton object.
- `Symbolic` is a class for operations that are the same for symbolic automata and symbolic transducers. This includes the operations that process the automata independently on the transition labels such as intersection or union.
- `Symbolic automata` is a class for operations over symbolic automata. It includes basic operations such as determinization and minimization as well as efficient and advanced ones such as simulations and antichain.
- `Symbolic Transducer` contains implementation of operations over symbolic transducers such as composition or application on NFA.
- `Finite automata` is a class containing optimised algorithms for classical finite automata.

Figure 6.1: Library Design



6.2 Predicates

The library supports two different types of predicates by default. When adding new types of predicates, predicate class and parser must be provided. The new predicates must implement the defined interface for the library to work correctly.

6.2.1 Default predicates

Predicates provided by the library as default are:

- *in* and *not_in* predicates, introduced in Section 2.5
- *letter* predicates representing symbols used in classic finite automata operations

6.2.2 Predicate interface

There are 5 operations needed for the implemented algorithms:

- conjunction — conjunction of two predicates
- disjunction — disjunction of two predicates
- negation — negation of a predicate
- satisfiability — check whether the predicate is satisfiable
- has_letter — check if a symbol belongs to the set represented by the predicate

Conjunction and *disjunction* are needed for basic algorithms such as intersection and union. Algorithms such as determinization and minimisation also need *complement* operation. *Satisfiability* is used in optimisations described in Section 3.5. The operation *has_letter* is used in advanced algorithms such as computing simulations relation and the antichains algorithm.

Chapter 7

Implementation

This chapter provides the description of the created library *symboliclib*. The more specific information about the library is given first. Then, information about the inner representation of different types of automata is given. Finally, implementation of various algorithms is discussed.

7.1 Symboliclib

Symboliclib is a library implemented in Python 3. It supports finite and symbolic automata as well as finite and symbolic transducers. The default input format of *symboliclib* is Timbuk [3] which is so far the only supported format. The library supports conversion of symbolic automata to classic finite automata and serialisation of automata back to Timbuk text format.

The predicates supported by default are *in* and *not_in* predicated inspired by FSA library[2] and *letter* predicates which represent symbols used in classic finite automata and transducers. Some of the algorithms which can be optimised to work on finite automata more efficiently than on symbolic automata are implemented for both types of automata separately.

Adding new types of predicates is simple as long as they implement the predefined interface. Usage of SMT solvers is also possible by creating a middle-layer that converts the interface of SMT solver to the interface required by *symboliclib*.

7.1.1 Command line interface

Symboliclib provides a simple command line interface written in bash. CLI takes a name of the desired operations and path to automata files as arguments. It should cover the most used automata operations as well as different algorithms for inclusion checking described in Section 7.4.

7.2 Transducers support

Symboliclib library has a basic support for transducers including operations such as union, intersection, composition or running transducer on NFA. Both classic finite and symbolic transducers are supported, but unlike automata, operations are not implemented separately for classic and symbolic transducers. Transducers containing epsilon transitions are not

supported, but used data structures and already implemented operations allow to add this support in the future.

Informally said, the only difference between automata and transducers input format and processing are the transition labels. Where one predicate is enough for automata to model configurations of systems, transducers use two predicates to model system behaviour. This difference is handled by *symboliclib* module `Transducer predicate`.

`Transducer predicate` provides a layer for transducer processing by executing predicate operations on both input and output predicates component-wise. This way, the interface for automata and transducer predicate processing is the same and algorithms such as intersection and union can be generalised to work on both automata and transducers.

7.3 Inner representation of automata

Inner representation of symbolic and finite automata and transducers is a Python class with the same basic attributes. The library does not support different input and output alphabets for transducers. The classes additionally include some advanced attributes used for optimisation such as *determinized* or *reversed* which represent the determinized and reversed version of the automaton. When determinizing or reversing, the library first checks whether these attributes are non-empty. If they are present, the desired modification of the automata is returned without the need to perform the operation again.

Attributes of the automata such as alphabets and initial, final and all states are represented by a set. This guarantees that the same state or symbol is not saved redundantly two times. As an optimisation, after parsing an automaton *symboliclib* removes all nonterminating and useless states and their respective transitions as described in Section 7.5.

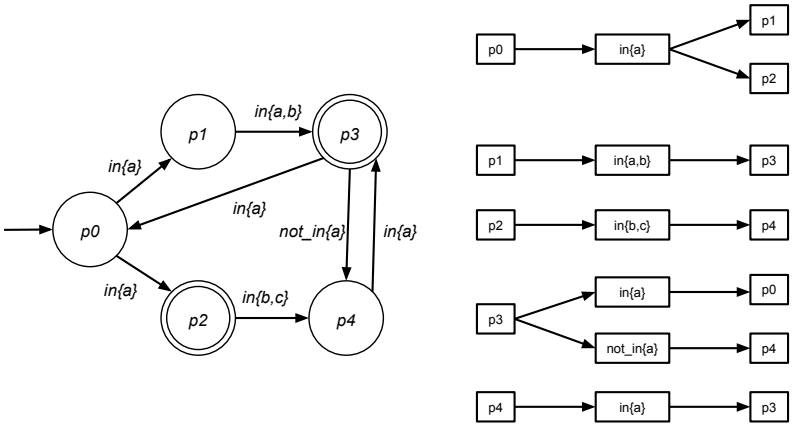
For easy manipulation with automata, transitions are saved in a list of dictionaries. The key of the dictionary is a left-handed side state of a transition and the value is another dictionary. In this dictionary, the key is a predicate and the value is a list of right-handed states.

For transitions $p0 \xrightarrow{in\{a,b\}} p1$, $p0 \xrightarrow{in\{a,b\}} p2$ and $p1 \xrightarrow{in\{b\}} p2$ the structure would be:

$$[\{\text{'p0'}: \{in\{a,b\} : [\text{p1,p2}]\}\}, \{\text{'p1'}: \{in\{b\} : [\text{p2}]\}]$$

A schematic example of automata transitions data structure can be found in Figure 7.1.

Figure 7.1: Automata transitions data structure and its internal representation



For an easier automata manipulation, the function *get_math_format* was added. This function returns automaton in a dictionary containing items *alphabet*, *states*, *initial*, *final* and *transitions*. This method should make possible to get automata in standard formal definition format and thus make easier usage for new users.

7.4 Algorithms

Most of the algorithms in *smyboliclib* (intersection, union, determinization, minimisation, epsilon rules removing, etc.) are based on Lecture notes from Javier Esparza [8].

Some of the operations such as intersection or simulations are separately implemented and optimised for classic finite automata. This allows avoiding operations on sets like intersection and union where a simple equality checking is sufficient. Generally, these operations should run faster on classic finite automata than on symbolic automata.

Symboliclib implements the following algorithms for language inclusion checking which is a widely used operation in formal verification:

- **Inclusion checking by mathematical definition** — *is_included_simple* — check whether $L(A) \subseteq L(B)$ by constructing $L(A) \cap \neg L(B)$
- **Classic inclusion checking** — *is_included* — based on [8]
- **Antichains** — *is_included_antichain_pure* — described in Section 4.3, without the simulations optimisation
- **Antichains with simulations** — *is_included_antichain* — described in Section 4.3, including the simulations optimisation

7.5 Optimizations

Symboliclib includes some optimisations for better performance and faster execution.

After parsing an input automaton, all nonterminating or unavailable states are removed from the automaton and so are all their corresponding transitions. By removing nonterminating and unavailable states, the language accepted by the automaton is not changed, but the size of the automaton may be decreased which leads to faster execution of operations.

The next optimisation of automata size is merging transition labels into one when possible. E.g., if an automaton contains transitions $p0 \xrightarrow{in\{a\}} p1$, $p0 \xrightarrow{in\{b\}} p1$, these are merged and saved as $p0 \xrightarrow{in\{a,b\}} p1$. This reduces the number of transitions of the automaton. The reduction is especially needed when the automaton was transformed from classic FA to symbolic, in which case each predicate represents only one symbol.

Chapter 8

Experimental Evaluation

This chapter describes the experimental evaluation of the language inclusion checking algorithms implemented in *symboliclib*. Possible reduction in automata size by using symbolic automata was also evaluated. The efficiency of symbolic automata operations in comparison with classic automata operations is discussed. Then, a comparison of an algorithm implemented in VATA and in *symboliclib* is given.

For the evaluations we used NFA from abstract regular model checking provided by VATA library. Since VATA does not support symbolic automata, the finite automata were transformed into equivalent symbolic automata so that the execution times are comparable. The tests were performed on a computer with Ubuntu 14.04 LTS, Intel(R) Core(TM) i3-3120M CPU (2,5 GHz, 2 cores, 256 K cache) and 4GB RAM.

As was expected, *symboliclib* is slower than VATA library. The inefficiency is partially caused by the differences in main goals of these libraries. VATA is created with the focus on fast and optimised automata processing, implemented in C++. On the other hand, *symboliclib* is written with the focus on easy and fast learning. A part of the inefficiency can also be caused by the implementation language Python, which is generally slower than C++.

8.1 Language inclusion

Symboliclib supports four different algorithms for language inclusion testing described in Section 7.4. In this evaluation, *simple inclusion* stands for inclusion checking by mathematical definition, *inclusion* for algorithm based on [8], *antichains* for antichains algorithm without the simulations optimisation and *simulations* for antichains algorithm with the simulations optimisation. The Figure 8.1 shows the comparison of them as well as the execution time in the VATA library. Average inclusion times can be found in Table 8.1. Language inclusion was tested on more than 3 000 pairs of classic finite automata with explicit encoding. The testing set included deterministic as well as nondeterministic automata, most of them having less than 3 000 transitions.

Generally, simple and classic inclusion checking should be more efficient for smaller or deterministic automata, when determinization is easy and fast or not needed at all. In such cases, the computation of simulations takes longer time than simple inclusion checking. The inclusion checking using antichains should be more efficient in nondeterministic automata that have a big number of states or transitions which can be exponentially increased during determinization.

The experimental evaluation results confirm the assumption that classic inclusion checking is efficient for smaller automata having less than 1 000 states. In these cases, the execution time in VATA library is 0.005 seconds and in *symboliclib* 0.151 seconds. The basic antichains algorithm beats all other *symboliclib* algorithms for any automata size.

On the other hand, antichains algorithm using simulations is the slowest for any automata size. Since the basic antichains algorithm is the fastest, the inefficiency is caused by computation of the simulation relation. For big automata, the chosen implementation of simulations is slow because of looping multiple times through the whole alphabet. A solution to this inefficiency would be to implement another algorithm for computing simulations relation or to optimise the current one.

Figure 8.1: A comparison of different algorithms for language inclusion checking in *symboliclib* with VATA. Evaluation was performed on classic finite automata with explicit encoding.

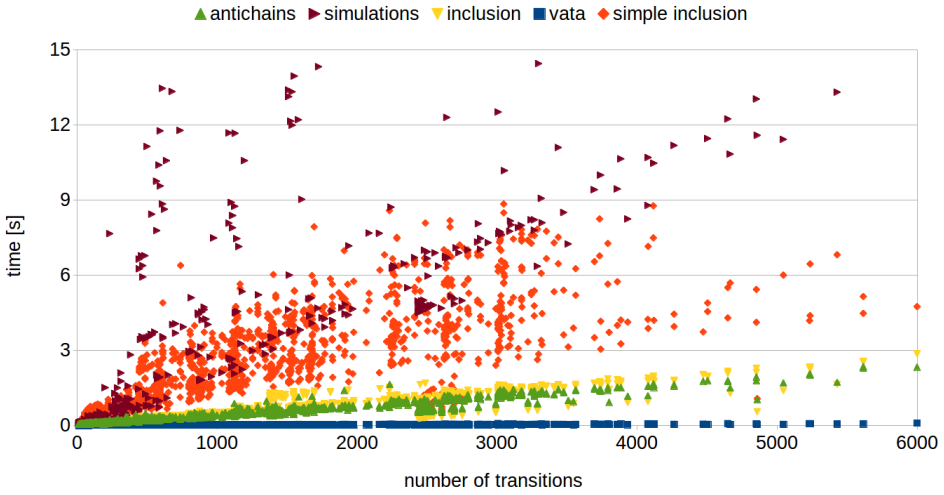


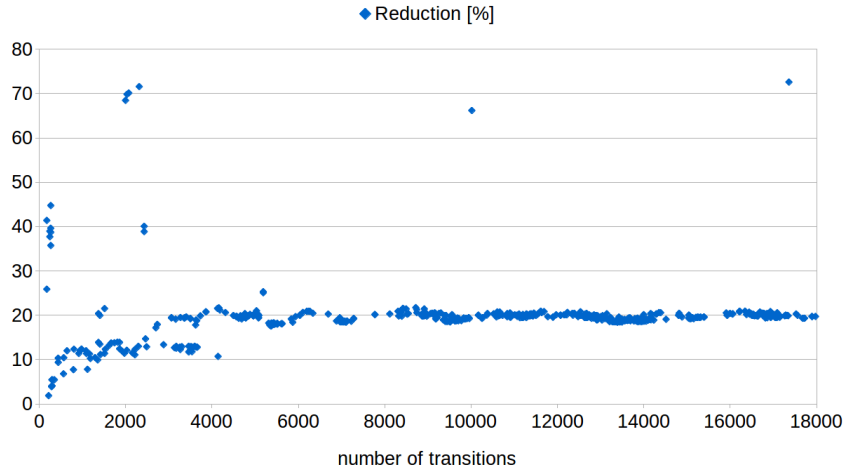
Table 8.1: Average language inclusion checking execution times (in seconds)

size	vata	simple inclusion	inclusion	antichains	simulations
0 - 1 000	0.005	0.598	0.151	0.130	1.004
1 000 - 2 000	0.020	2.993	0.743	0.559	7.485
2 000 - 3 000	0.028	3.360	0.896	0.871	7.247
3 000 - 4 000	0.032	4.980	1.409	1.254	8.486
4 000 - 5 000	0.049	4.861	1.772	1.613	11.717
5 000 - 6 000	0.050	5.270	2.227	2.056	14.006

8.2 Size reduction using symbolic automata

The biggest advantage of symbolic automata is the possibility of uniting multiple transitions into one by using predicates instead of symbols. The average size reduction in the number of transitions was tested on a set of more than 1 200 automata with 10 to 18 000 transitions. The Figure 8.2 shows the decrease in the number of automata transitions.

Figure 8.2: Decrease in automata size using symbolic automata



The decrease should be higher for automata which use more symbols on transitions between the same states. In this case the symbols are represented by one predicate and all such transitions are replaced with one transition. The size is not reduced if such transitions do not exist.

As we see in the Figure 8.2, an average automaton size is decreased by 20% when using symbolic automata. For automata having less than 4 000 transitions, the reduction is smaller, usually 10%. In some of the cases, the reduction can be as high as 70%.

Eliminating 20% of transitions without changing the automata language is very useful for big automata. Additionally, symbolic automata support most of the algorithms for classic letter automata as was shown in Chapter 3. Therefore, using symbolic automata instead of classic letter automata is useful as it leads to decrease in automata size without lowering the number of available operations.

8.3 Symbolic automata operations

This section provides comparison of VATA intersection with intersection for letter and symbolic automata in *symboliclib*. The results can be seen in Figure 8.3 and Table 8.2. As was expected, VATA is faster than *symboliclib*, but for automata having less than 5 000 states the difference is not so notable.

We can observe that processing classic automata takes generally less time than the same operation on equivalent symbolic automata. This fact supports the assumption that reimplementing some of the automata operations for classic automata increases the efficiency of the library. This is because the predicates in symbolic automata represent a set of symbols and therefore the intersection and union of such predicates include operations over sets in Python. On the other hand, classic finite automata work with symbols, where intersection and union may be replaced by simple equality which is a faster operation than operations over sets.

As was shown in the previous section, using symbolic automata can decrease the automata size by 20%. The higher execution time of symbolic automata operations is a trade-off to this reduction.

Figure 8.3: Automata intersection execution times

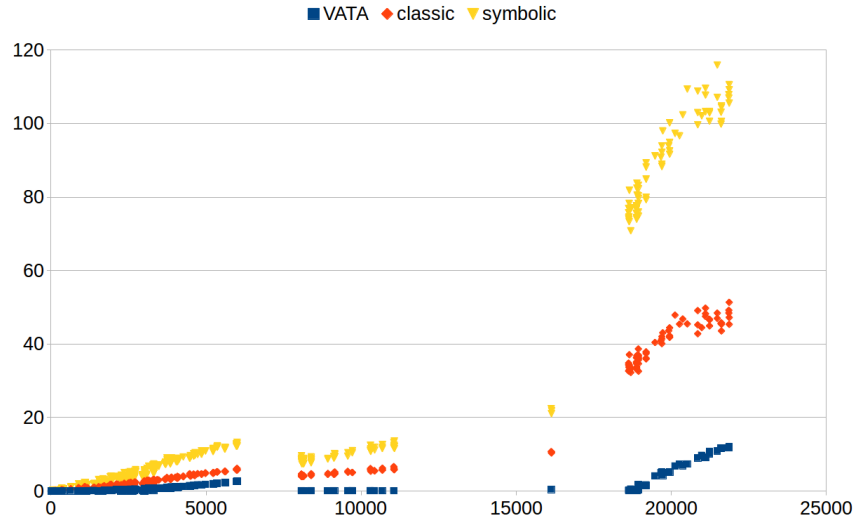


Table 8.2: Average automata intersection execution times (in seconds)

size	vata	classic	symbolic
0 - 5 000	0.252	1.544	3.372
5 000 - 10 000	1.116	4.869	10.221
10 000 - 15 000	0.066	5.895	11.891
15 000 - 20 000	1.663	35.270	78.516
20 000 - 25 000	8.813	45.521	102.165

8.4 Easy Prototyping

An example of implementation in *symboliclib* in comparison with VATA is given to show simplicity of prototyping in *symboliclib*.

The simplicity of programming in *symboliclib* can be shown on the complement of automata. Code for this algorithm in *symboliclib* can be seen in Algorithm 9 and in VATA in Algorithm 10. Some unimportant parts of the code for this comparison such as comments have been skipped.

In *symboliclib*, the automaton is first determinized and then the final states are exchanged for nonfinal states through set operations. This process comes directly from the definition of complement of FA so it is straightforward and easy-to-understand.

In VATA, complement requires looping through the automata states and checking whether they are final or nonfinal. The constructing of complement is less clear because of multiple nested loops. Additionally, the determinization is not implicit, so the user has to remember to determinize the automaton first and construct the complement after that.

The difference in implementation is partially caused by the different programming languages. VATA is implemented in C++, which is generally more complicated language, as a trade-off to efficiency. On the other hand, *symboliclib* is written in Python, which has simpler syntax, but is slower.

Algorithm 9: Automata complement algorithm in symboliclib

Input: NFA $self = (\Sigma, Q_A, I_A, F_A, \delta_A)$
Output: NFA $complement = (\Sigma, Q_B, I_B, F_B, \delta_B)$; $L_{complement} = \Sigma^* - L_{self}$

```
1 det = self.determinize();
2 complement = deepcopy(det);
3 complement.final = det.states - det.final;
4 return complement;
```

Algorithm 10: Automata complement algorithm in VATA

Input: NFA $aut = (\Sigma, Q_A, I_A, F_A, \delta_A)$
Output: NFA $res = (\Sigma, Q_B, I_B, F_B, \delta_B)$; $L_{res} = \Sigma^* - L_{aut}$

```
1  ExplicitFA res;
2  for auto stateToCluster : *transitions do
3      if !aut.IsStateFinal(stateToCluster.first) then
4          res.SetStateFinal(stateToCluster.first);
5      end
6      for auto symbolToSet : *stateToCluster.second do
7          for auto stateInSet : symbolToSet.second do
8              if !aut.IsStateFinal(stateInSet) then
9                  res.SetStateFinal(stateInSet);
10             end
11         end
12     end
13 end
14 return res
```

Chapter 9

Conclusion

The goal of this thesis was to create an automata library suitable for fast prototyping of new algorithms. Therefore defined data structures are transparent and easy-to-understand. The library supports simple adding new types of predicates and new algorithms.

The created library *symboliclib* supports classic finite and symbolic automata as well as transducers. Some state-of-the-art algorithms have been implemented including simulations relation computation and language inclusion checking using antichains. The library was implemented in Python3 which is a popular language with relatively simple syntax. The input format of the library is Timbuk which is also used in other automata libraries such as VATA. *Symboliclib* implements basic operations over automata and transducers such as intersection, union, determinization or minimalization. Language inclusion checking has been implemented using four different algorithms.

The library was tested on set of automata provided by the supervisor of this thesis. The results indicate that while the library is slower than other existing optimised libraries, it works correctly and can be used for predesigned purposes. A trade off to this inefficiency is transparent design and fast learning curve which makes prototyping and testing of new algorithms possible. *Symboliclib* is roughly 8 times slower than VATA on automata having less than 15 000 transitions but the execution time grows exponentially. This fact does not limit testing of new algorithms using the *symboliclib* library in any significant way.

The further development could be more operations over transducer. The library could be modified to support transducers with epsilon transitions. For further optimisation of the library, a profilation could be done for detection of slow algorithms. These algorithms can then be optimised which will add up to the efficiency. Some state-of-the-art algorithms such as simulation relation could be further optimised or implemented in other ways. As was determined in the evaluation, operations over classic finite automata take significantly lower time than on symbolic automata so some of the operations currently implemented only for symbolic automata can be modified and implemented for classic finite automata.

As another optimisations, simulations and antichains could be altered to work faster on symbolic automata. Currently, the implementation of these operations is simple and yields correct results. On the other hand, it is not very efficient since it basically transforms symbolic automata transitions to classic finite automata. The algorithms could be designed in another way to work directly with the predicates instead of checking all symbols of automata alphabet.

Currently, *symboliclib* is not very optimised, and thus it is not the best choice for performance critical applications. However, *symboliclib* is suitable for new automata algorithms implementation testing or for study purposes.

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Appendix A

Storage Medium

The storage medium contains the sources of created library. It also contains an electronic version of this text report.